# EE 2200 INTRO. TO ELECTRONIC DEVICES 

## ECE, Missouri S\&T

@ 2024 Steve E. Watkins

# WORKSHEETS FOR <br> EXAMINATION 1 

Topics Include:<br>Definitions<br>Semiconductor Crystals<br>Carriers and Doping (Resistivity)<br>Drift and Diffusion Currents and Junction<br>Diodes and Diode Circuits

Boltzmann's constant:
Planck's constant
Electronic charge:
kT at 300 K
Free-space permittivity
Relative permittivity
Bandgap energies
$\mathrm{k}=1.381 \times 10^{-23} \mathrm{~J} / \mathrm{K}=8.618 \times 10^{-5} \mathrm{eV} / \mathrm{K}$
$\mathrm{h}=4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}=6.626 \times 10^{-34} \mathrm{~J}-\mathrm{sec}$
$\mathrm{q}=1.602 \times 10^{-19} \mathrm{C}$
$\mathrm{kT}=0.0259 \mathrm{eV} \quad$ eV-J conversion $\quad 1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$
$\varepsilon_{\mathrm{O}}=8.854 \times 10^{-14} \mathrm{~F} / \mathrm{cm} \quad$ Speed of Light $\quad \mathrm{c}=2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s}$
Si: $11.9 \quad$ Ge: 16.0
Si: $1.12 \mathrm{eV} \quad$ Ge: 0.67 eV

GaAs: 13.1
GaAs: 1.42 eV

## Semiconductor Example

Consider a Si crystal which is doped with $10^{12}$ phosphorous $(\mathrm{P})$ atoms $/ \mathrm{cm}^{3}$ and with $10^{13}$ aluminum $(\mathrm{Al})$ atoms $/ \mathrm{cm}^{3}$. Assume all of the dopants are ionized and that the semiconductor is at room temperature.

Identify the dopants as donors and/or acceptors.
Host Si (col IV). P is col V, hence it is a donor
Al is col III, hence it is an acceptor.

## Calculate the equilibrium electron and hole concentrations.

$\left(\mathrm{Na}_{\mathrm{a}}{ }^{-}>\mathrm{N}_{\mathrm{d}}{ }^{+}\right.$), hence the majority carrier will be holes
The equations are

$$
\mathrm{n}_{0}+\mathrm{N}_{\mathrm{a}}^{-}=\mathrm{p}_{0}+\mathrm{N}_{\mathrm{d}}^{+} \quad \mathrm{n}_{0} \mathrm{p}_{0}=\mathrm{n}_{\mathrm{i}}^{2}
$$

Solve for the majority carrier $\mathrm{p}_{0}$. Substitute for a single equation in $\mathrm{p}_{0}$.

$$
\begin{array}{lll}
\mathrm{p}_{0}-\left(\mathrm{Na}^{-}-\mathrm{N}_{\mathrm{d}}+\right)-\mathrm{n}_{0}=0 & \mathrm{n}_{0}=\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{p}_{0} \\
\mathrm{p}_{0}-\left(\mathrm{Na}^{-}-\mathrm{N}_{\mathrm{d}}^{+}\right)-\left(\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{p}_{0}\right)=0 & \text { or } & \mathrm{p}_{0}^{2}-\left(0.9 \times 10^{13}\right) \mathrm{p}_{0}-\left(\mathrm{n}_{\mathrm{i}}^{2}\right)=0
\end{array}
$$

Taking the physical solution of this quadratic equation.

$$
\begin{aligned}
& \mathrm{p}_{0}=-(1 / 2)\left[-\left(0.9 \times 10^{13}\right)+(1 / 2) \sqrt{ }\left\{\left[-\left(0.9 \times 10^{13}\right)\right]^{2}-4\left(-\mathrm{n}_{\mathrm{i}}^{2}\right)\right\}\right. \\
& \mathrm{p}_{0}=(1 / 2)\left(0.9 \times 10^{13}\right)+(1 / 2) \sqrt{ }\left(\left(0.9 \times 10^{13}\right)^{2}+4\left(1.5 \times 10^{10}\right)^{2}\right] \\
& \mathrm{p}_{0}=9.00 \times 10^{12} \mathrm{~cm}^{-3}\left(=9.000025 \times 10^{12} \mathrm{~cm}^{-3}\right)
\end{aligned}
$$

The minority carrier concentration is

$$
\mathrm{n}_{0}=\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{p}_{0}=\left(1.5 \times 10^{10}\right)^{2} /\left(9.00 \times 10^{12}\right)=2.50 \times 10^{7} \mathrm{~cm}^{-3}
$$

## Calculate the Fermi level.

The equation in terms of $\mathrm{p}_{\mathrm{o}}$ is

$$
\begin{aligned}
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right)=-\mathrm{kT} \operatorname{In}\left(\mathrm{p}_{\mathrm{o}} / \mathrm{n}_{\mathrm{i}}\right) \\
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right)=-(0.0259 \mathrm{eV}) \ln \left[\left(9.00 \times 10^{12}\right) /\left(1.5 \times 10^{10}\right)\right]=-0.1657 \mathrm{eV}
\end{aligned}
$$

Calculate the resistivity given mobilities of $\mu_{n}=1450 \mathrm{~cm}^{2} / V_{s}$ and $\mu_{p}=500 \mathrm{~cm}^{2} / / \mathrm{Vs}$.

$$
\begin{aligned}
& \rho=1 / \sigma=\left[\mathrm{q}\left(\mathrm{n}_{0} \mu_{\mathrm{n}}+\mathrm{p}_{0} \mu_{\mathrm{p}}\right)\right]^{-1} \\
& \rho=1 /\left\{\left(1.602 \times 10^{-19}\right)\left[\left(2.50 \times 10^{7}\right)(1450)+\left(9.00 \times 10^{12}\right)(500)\right]\right\} \\
& \rho=1,387(\Omega-\mathrm{cm})
\end{aligned}
$$

## Semiconductor Example

Consider a Si crystal which is doped with $10^{12}$ phosphorous $(\mathrm{P})$ atoms $/ \mathrm{cm}^{3}$ and with $10^{13}$ aluminum $(\mathrm{Al})$ atoms $/ \mathrm{cm}^{3}$. Assume all of the dopants are ionized and that the semiconductor is at room temperature.

Identify the dopants as donors and/or acceptors.

Calculate the equilibrium electron and hole concentrations.

Calculate the Fermi level.

Calculate the resistivity given mobilities of $\mu_{n}=1450 \mathrm{~cm}^{2} / V s$ and $\mu_{p}=500 \mathrm{~cm}^{2} / V \mathrm{Vs}$.

## Semiconductor Example

Consider a Si crystal with a Fermi level of $\left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right)=0.200 \mathrm{eV}$. The dimensions are length 5 mm , width 1 mm , and height 1 mm . Assume that the semiconductor is at room temperature.

Is the semiconductor n-type or p-type and why?
The semiconductor is n -type since the Fermi level is positive (e.g. above $\mathrm{E}_{\mathrm{i}}$ ).
Calculate the equilibrium electron and hole concentrations.
The equation in terms of $\mathrm{n}_{0}$ is

$$
\begin{aligned}
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right)=\mathrm{kT} \operatorname{In}\left(\mathrm{n}_{\mathrm{o}} / \mathrm{n}_{\mathrm{i}}\right) \text { or } \mathrm{n}_{\mathrm{o}}=\mathrm{n}_{\mathrm{i}} \exp \left[\left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right) / \mathrm{kT}\right] \\
& \mathrm{n}_{\mathrm{o}}=\left(1.5 \times 10^{10}\right) \exp \left[(0.200 \mathrm{eV}) /(0.0259 \mathrm{eV}]=3.39 \times 10^{13} \mathrm{~cm}^{-3}\right.
\end{aligned}
$$

Then,

$$
\mathrm{p}_{0}=\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{n}_{0}=\left(1.5 \times 10^{10}\right)^{2} /\left(3.39 \times 10^{13}\right)=6.64 \times 10^{6} \mathrm{~cm}^{-3}
$$

## What are the doping concentrations?

Insufficient information is given to calculate the doping concentrations.
However, the effective doping concentration from $\mathrm{n}_{0}+\mathrm{N}_{\mathrm{a}}{ }^{-}=\mathrm{p}_{0}+\mathrm{N}_{\mathrm{d}}{ }^{+}$is

$$
\left(\mathrm{N}_{\mathrm{d}}{ }^{+}-\mathrm{Na}^{-}\right)=\mathrm{n}_{0}-\mathrm{p}_{0} \sim 3.39 \times 10^{13} \mathrm{~cm}^{-3}
$$

Calculate the resistivity given mobilities of $\mu_{n}=1450 \mathrm{~cm}^{2} / \mathrm{Vs}$ and $\mu_{p}=500 \mathrm{~cm}^{2} / \mathrm{Vs}$.

$$
\begin{aligned}
& \rho=1 / \sigma=\left[q\left(\mathrm{n}_{0} \mu_{\mathrm{n}}+\mathrm{p}_{0} \mu_{\mathrm{p}}\right)\right]^{-1} \\
& \rho=1 /\left\{\left(1.602 \times 10^{-19}\right)\left[\left(3.39 \times 10^{13}\right)(1450)+\left(6.64 \times 10^{6}\right)(500)\right]\right\} \\
& \rho=1 / 0.007866=127(\Omega-\mathrm{cm})
\end{aligned}
$$

Calculate the resistance.
The resistance is

$$
\begin{aligned}
& \mathrm{R}=(\mathrm{L} / \mathrm{A}) \rho=[0.5 \mathrm{~cm} /(0.1 \mathrm{~cm})(0.1 \mathrm{~cm})] 127(\Omega-\mathrm{cm})= \\
& \mathrm{R}=6,357 \Omega
\end{aligned}
$$

## Semiconductor Example

Consider a Si crystal with a Fermi level of $\left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{i}}\right)=0.200 \mathrm{eV}$. The dimensions are length 5 mm , width 1 mm , and height 1 mm . Assume that the semiconductor is at room temperature.

Is the semiconductor n-type or p-type and why?

Calculate the equilibrium electron and hole concentrations.

What are the doping concentrations?

Calculate the resistivity given mobilities of $\mu_{n}=1450 \mathrm{~cm}^{2} / V s$ and $\mu_{p}=500 \mathrm{~cm}^{2} / \mathrm{Vs}$.

Calculate the resistance.

## Junction Example

Consider abrupt pn junctions made of Si and Ge . Assume RT. Doping values are $\mathrm{Nap}_{\mathrm{ap}}{ }^{-}=10^{16} \mathrm{~cm}^{-3}$ and $\mathrm{N}_{\mathrm{dp}}{ }^{+}=0$ on the p side and $\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{16} \mathrm{~cm}^{-3}$ and $\mathrm{Nan}^{-}=0$ on the n side.

Calculate the contact potential $V_{o}$ values in volts and energy difference $q V_{o}$ values in eV for Si and Ge.

For extrinsic doping, the contact potential is
$\mathrm{V}_{0}=(\mathrm{kT} / \mathrm{q}) \ln \left[\left(\mathrm{Nap}^{-}-\mathrm{N}_{\mathrm{dp}}{ }^{+}\right)\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}-\mathrm{Nan}^{-}{ }^{-}\right) / \mathrm{n}_{\mathrm{i}}{ }^{2}\right]$
Note that $\mathrm{kT}=0.0259 \mathrm{eV}, 1 \mathrm{C} \times 1 \mathrm{~V}=1 \mathrm{~J}$, and $1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$, then

$$
\mathrm{kT} / \mathrm{q}=\left[(0.0259 \mathrm{eV}) \times\left(1.602 \times 10^{-19} \mathrm{~J} / 1 \mathrm{eV}\right)\right] \times\left(1 / 1.602 \times 10^{-19} \mathrm{C}\right)
$$

$$
\mathrm{kT} / \mathrm{q}=0.0259 \mathrm{~J} / \mathrm{C}=\mathbf{0 . 0 2 5 9} \mathrm{V}
$$

For a Silicon Junction,

$$
\begin{aligned}
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \ln \left[\left(10^{16}-0\right)\left(10^{16}-0\right) /\left(1.5 \times 10^{10}\right)^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \operatorname{In}\left[4.444 \times 10^{11}\right]=\mathbf{0 . 6 9 5} \mathrm{V} \\
& \mathrm{qV}_{0}=\left\{\left[\left(1.602 \times 10^{-19}\right) \times(0.695)\right] \mathrm{J}\right\} \times\left(1 \mathrm{eV} / 1.602 \times 10^{-19} \mathrm{~J}\right)=\mathbf{0 . 6 9 5} \mathbf{e V}
\end{aligned}
$$

For a Germanium Junction,

$$
\begin{aligned}
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \ln \left[\left(10^{16}-0\right)\left(10^{16}-0\right) /\left(2.3 \times 10^{13}\right)^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \operatorname{In}\left[1.89 \times 10^{5}\right]=\mathbf{0 . 3 1 5 ~ V} \\
& \mathrm{qV}_{0}=\left\{\left[\left(1.602 \times 10^{-99}\right) \times(0.315)\right] \mathrm{J}\right\} \times\left(1 \mathrm{eV} / 1.602 \times 10^{-9} \mathrm{~J}\right)=\mathbf{0 . 3 1 5} \mathbf{e V}
\end{aligned}
$$

State the trend contact potential with respect to $E_{G}$.
Si $\mathrm{E}_{\mathrm{G}}=1.12 \mathrm{eV}$ and $\mathrm{Ge}_{\mathrm{G}}=0.67 \mathrm{eV}$
As $E_{G}$ increases (and $n_{i}$ decreases), the contact potential $V_{0}$ increases.
For abrupt pn junctions with $\left(N_{a p}{ }^{-}-N_{d p}{ }^{+}\right)=\left(N_{d n}{ }^{+}-N_{a n}{ }^{-}\right)$, sketch the equilibrium band diagram including the Fermi level, $E_{i p}, E_{i n}$, and the energy difference $q V_{o}$.


## Junction Example

Consider abrupt pn junctions made of Si and Ge . They are doped with

$$
\mathrm{N}_{\mathrm{ap}}{ }^{-}=10^{16} \mathrm{~cm}^{-3} \text { and } \mathrm{N}_{\mathrm{dp}}{ }^{+}=0 \text { on the } \mathrm{p} \text { side and }
$$

$\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{16} \mathrm{~cm}^{-3}$ and $\mathrm{Nan}^{-}=0$ on the n side.
Calculate the contact potential $V_{o}$ values in volts and energy difference $q V_{o}$ values in eV for Si and Ge.

State the trend contact potential with respect to $E_{G}$.

For abrupt pn junctions with $\left(N_{a p}{ }^{-}-N_{d p}{ }^{+}\right)=\left(N_{d n}{ }^{+}-N_{a n}{ }^{-}\right)$, sketch the equilibrium band diagram including the Fermi level, $E_{i p}, E_{i n}$, and the energy difference $q V_{o}$.

## Junction Example

Consider an abrupt Si junction with only $\mathrm{Nap}^{-}=10^{17} \mathrm{~cm}^{-3}$ on the p side and only $\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{13} \mathrm{~cm}^{-3}$ on the n side. Assume that the semiconductor is at room temperature.

## Calculate the contact potential.

For extrinsic doping, the contact potential is

$$
\begin{aligned}
& \mathrm{V}_{0}=(\mathrm{kT} / \mathrm{q}) \ln \left[\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}-\mathrm{N}_{\mathrm{dp}}^{+}\right)\left(\mathrm{N}_{\mathrm{dn}^{+}}-\mathrm{Nanan}^{-}\right) / \mathrm{n}_{\mathrm{i}}^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \ln \left[\left(10^{17}-0\right)\left(10^{13}-0\right) /\left(1.5 \times 10^{10}\right)^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \operatorname{In}\left[4.444 \times 10^{9}\right] \\
& \mathrm{V}_{0}=0.575 \mathrm{~V}
\end{aligned}
$$

Calculate the Fermi levels.
For the p side

$$
\begin{aligned}
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{ip}}\right)=-\mathrm{kT} \operatorname{In}\left(\mathrm{p}_{\mathrm{o}} / \mathrm{n}_{\mathrm{i}}\right) \\
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{ip}}\right)=-(0.0259 \mathrm{eV}) \ln \left[\left(10^{17}\right) /\left(1.5 \times 10^{10}\right)\right]=-0.4070 \mathrm{eV}
\end{aligned}
$$

For the n side

$$
\begin{aligned}
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{in}}\right)=\mathrm{kT} \operatorname{In}\left(\mathrm{o} / \mathrm{n}_{\mathrm{i}}\right) \\
& \left(\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{in}}\right)=(0.0259 \mathrm{eV}) \ln \left[\left(10^{13}\right) /\left(1.5 \times 10^{10}\right)\right]=0.1684 \mathrm{eV}
\end{aligned}
$$

Note that

$$
\begin{aligned}
& \left(E_{i p}-E_{\text {in }}\right)=-\left(E_{F}-E_{i p}\right)+\left(E_{F}-E_{\text {in }}\right)=0.407+0.168=0.575 \mathrm{eV} \\
& \left(\mathrm{E}_{\text {ip }}-\mathrm{E}_{\text {in }}\right)=\mathrm{qV} \mathrm{~V}_{\mathrm{o}}=0.575 \mathrm{eV}
\end{aligned}
$$

Note if $\mathrm{V}_{0}=\mathrm{X}($ in V$)$,

$$
\text { then } \mathrm{qV}_{\mathrm{o}}=\left(1.602 \times 10^{-19}\right)(\mathrm{X}) \mathrm{J}
$$

$$
\text { and } \mathrm{qV}_{\mathrm{o}}=\left(1.602 \times 10^{-19}\right)(\mathrm{X}) \mathrm{J}(1 \mathrm{eV}) /\left(1.602 \times 10^{-19}\right)=\mathrm{XeV}
$$

Calculate the ratio of $x_{n o} / x_{p o}$.
The charge density relationship is

$$
\begin{aligned}
& \left|Q-\left|=\left|\mathrm{Q}_{+}\right| ; \quad\left(\mathrm{Nap}_{\mathrm{ap}}{ }^{-}-\mathrm{N}_{\mathrm{dp}}{ }^{+}\right) A \mathrm{x}_{\mathrm{p} 0}=\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}-\mathrm{Nan}^{-}\right) \mathrm{Ax}_{\mathrm{n} 0}\right. \text {; }\right. \\
& \text { or } \quad\left(\mathrm{N}_{\text {ap }}\right)_{\text {Eff }} \mathrm{X}_{\mathrm{p} 0}=\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}\right)_{\text {Eff }} \mathrm{X}_{\mathrm{n} 0} \text {; } \\
& \mathrm{X}_{\mathrm{n} 0} / \mathrm{X}_{\mathrm{p} 0}=\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}\right)_{\mathrm{Eff}} /\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}}=\left(10^{17}-0\right) /\left(10^{13}-0\right)=10,000
\end{aligned}
$$

## Junction Example

Consider an abrupt Si junction with only $\mathrm{Nap}^{-}=10^{17} \mathrm{~cm}^{-3}$ on the p side and only $\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{13} \mathrm{~cm}^{-3}$ on the n side. Assume that the semiconductor is at room temperature.

Calculate the contact potential.

Calculate the Fermi levels.

Calculate the ratio of $x_{n o} / x_{p o}$.

## Junction Example

Consider an abrupt Si junction with $\mathrm{Nap}^{-}=10^{16} \mathrm{~cm}^{-3}$ and $\mathrm{N}_{\mathrm{dp}}{ }^{+}=10^{15} \mathrm{~cm}^{-3}$ on the p side and only $\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{15} \mathrm{~cm}^{-3}$ on the n side. Assume that the semiconductor is at room temperature.

## Calculate the contact potential.

For extrinsic doping, the contact potential is

$$
\begin{aligned}
& \mathrm{V}_{0}=(\mathrm{kT} / \mathrm{q}) \ln \left[\left(\mathrm{Nap}_{\mathrm{ap}}{ }^{-}-\mathrm{N}_{\mathrm{dp}}{ }^{+}\right)\left(\mathrm{N}_{\mathrm{dn}^{+}}-\mathrm{N}_{\mathrm{an}}{ }^{-}\right) / \mathrm{n}_{\mathrm{i}}^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \ln \left[\left(10^{16}-10^{15}\right)\left(10^{15}-0\right) /\left(1.5 \times 10^{10}\right)^{2}\right] \\
& \mathrm{V}_{0}=(0.0259 \mathrm{~V}) \operatorname{In}\left[4.000 \times 10^{10}\right] \\
& \mathrm{V}_{0}=0.632 \mathrm{~V}
\end{aligned}
$$

Calculate the depletion width $W_{0}$ at equilibrium.
For equilibrium, the depletion width is (note for $\operatorname{Si} \varepsilon_{\mathrm{r}}=11.8$ )

$$
\begin{aligned}
& \mathrm{W}_{0}=\left\{\left[2 \varepsilon_{\mathrm{r}} \varepsilon_{0}\left(\mathrm{~V}_{0}\right) / \mathrm{q}\right]\left[\left(\mathrm{N}_{\mathrm{ap}}\right)_{\mathrm{Eff}}+\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}}\right] /\left[\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}\right)_{\mathrm{Eff}}\left(\mathrm{~N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}}\right]\right\}^{1 / 2} \\
& \mathrm{~W}_{0}=\left\{\left[2(11.8)\left(8.854 \times 11^{-14}\right)(0.632) /\left(1.602 \times 10^{-1}\right)\right.\right. \\
&\left.\quad\left[\left(10^{16}-10^{15}\right)+\left(10^{15}-0\right)\right] /\left[\left(10^{16}-10^{15}\right)\left(10^{15}-0\right)\right]\right\}^{1 / 2} \\
& \mathrm{~W}_{0}= 0.957 \times 10^{-4} \mathrm{~cm}=0.957 \mu \mathrm{~m}
\end{aligned}
$$

Calculate the equilibrium depletion values $x_{n o}$ and $x_{p o}$.
The charge density relationship is $\left|\mathrm{Q}_{-}\right|=\left|\mathrm{Q}_{+}\right|$

Then $\mathrm{W}_{0}=\mathrm{x}_{\mathrm{p} 0}+\mathrm{x}_{\mathrm{n} 0}=\mathrm{x}_{\mathrm{p} 0}+9 \mathrm{x}_{\mathrm{p} 0}=10 \mathrm{x}_{\mathrm{p} 0}$

$$
\mathrm{x}_{\mathrm{p} 0}=(1 / 10) \mathrm{W}_{\mathrm{o}}=0.0957 \mu \mathrm{~m}
$$

$$
\mathrm{x}_{\mathrm{n} 0}=9 \mathrm{x}_{\mathrm{p} 0}=0.862 \mu \mathrm{~m}
$$

Calculate the depletion width $W$ for a reverse bias of $V=-100 \mathrm{~V}$.
For the junction under bias, the depletion width is (note for $\mathrm{Si} \varepsilon_{\mathrm{r}}=11.8$ )

$$
\begin{aligned}
\mathrm{W} & =\left\{\left[2 \varepsilon_{\mathrm{r}} \varepsilon_{0}\left(\mathrm{~V}_{0}-\mathrm{V}\right) / \mathrm{q}\right]\left[\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}\right)_{\mathrm{Eff}}+\left(\mathrm{N}_{\mathrm{dn}}\right)_{\mathrm{Eff}}\right] /\left[\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}\right)_{\mathrm{Eff}}\left(\mathrm{~N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}}\right]\right\}^{1 / 2} \\
\mathrm{~W} & =\left\{\left[2(11.8)\left(8.854 \times 10^{-14}\right)(0.632+100) /\left(1.602 \times 10^{-19}\right)\right]\right. \\
\mathrm{W} & \left.\left.\left.=12.1 \times 10^{-4}-10^{15}\right)+\left(10^{15}-0\right)\right] /\left[\left(10^{16}-10^{15}\right)\left(10^{15}-0\right)\right]\right\}^{1 / 2} \\
& {[(12.1}
\end{aligned}
$$

$$
\begin{aligned}
& \left(\mathrm{Nap}_{\mathrm{ap}}{ }^{-}-\mathrm{N}_{\mathrm{dp}}{ }^{+}\right) \mathrm{Ax}_{\mathrm{p} 0}=\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}-\mathrm{Nan}_{\mathrm{an}}{ }^{-}\right) \mathrm{Ax} \mathrm{x}_{\mathrm{n} 0} \text { or } \quad\left(\mathrm{N}_{\mathrm{ap}}{ }^{-}\right)_{\mathrm{Eff}} \mathrm{X}_{\mathrm{p} 0}=\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}} \mathrm{X}_{\mathrm{n} 0} \\
& \mathrm{x}_{\mathrm{n} 0}=\left[\left(\mathrm{Nap}_{\mathrm{ap}}\right)_{\mathrm{Eff}} /\left(\mathrm{N}_{\mathrm{dn}}{ }^{+}\right)_{\mathrm{Eff}}\right] \mathrm{x}_{\mathrm{p} 0}=\left[\left(10^{16}-10^{15}\right) /\left(10^{15}-0\right)\right] \mathrm{x}_{\mathrm{p} 0}=9 \mathrm{x}_{\mathrm{p} 0}
\end{aligned}
$$

## Junction Example

Consider an abrupt Si junction with $\mathrm{Nap}^{-}=10^{16} \mathrm{~cm}^{-3}$ and $\mathrm{N}_{\mathrm{dp}}{ }^{+}=10^{15} \mathrm{~cm}^{-3}$ on the p side and only $\mathrm{N}_{\mathrm{dn}}{ }^{+}=10^{15} \mathrm{~cm}^{-3}$ on the n side. Assume that the semiconductor is at room temperature.

Calculate the contact potential.

Calculate the depletion width $W_{0}$ at equilibrium.

Calculate the equilibrium depletion values $x_{\text {no }}$ and $x_{p o}$.

Calculate the depletion width $W$ for a reverse bias of $V=-100 \mathrm{~V}$.

## Diode Biasing Example

Consider the following circuit with a source voltage $\mathrm{V}_{\mathrm{S}}=5.0 \sin (10 \mathrm{t}) \mathrm{V}$, a load resistor $\mathrm{R}=1.00 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}$ and reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$.


Calculate the maximum voltage and current for forward bias, i.e. the diode is on.
The diode is forward biased when $\mathrm{V}_{\mathrm{S}}=+5 \mathrm{~V}$
For forward bias (away from the knee of the IV curve), the diode voltage is

$$
\mathrm{V}_{\text {max }}=\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}
$$

By the load line equation (from KVL) the current is

$$
\mathrm{I}_{\max }=(1 / \mathrm{R})\left[\mathrm{V}_{\mathrm{s}}-\left(\mathrm{V}_{\text {to }}\right)\right]=(1 / 1000)[5.0-0.7]=4.3 \mathrm{~mA}
$$

Calculate the voltage across the resistor for this maximum condition.
By Ohm's Law

$$
\mathrm{V}_{\mathrm{R} \max }=\mathrm{I}_{\max } \mathrm{R}=(4.3 \mathrm{~mA})(1000 \Omega)=4.3 \mathrm{~V}
$$

Calculate the minimum voltage and current for reverse bias, i.e. the diode is off.
The diode is reverse biased when $V_{S}=-5 \mathrm{~V}$
For reverse bias (away from the knee of the IV curve), the diode current is

$$
\mathrm{I}_{\min }=-\mathrm{I}_{0}=-0.01 \mathrm{~mA}
$$

By the load line equation (from KVL) the voltage is

$$
\begin{aligned}
& \mathrm{V}_{\min }=\mathrm{V}_{\mathrm{s}}-\left(\mathrm{I}_{\min }\right) \mathrm{R}=\mathrm{V}_{\mathrm{s}}-\left(-\mathrm{I}_{0}\right) \mathrm{R}=-5.0-(-0.01 \mathrm{~mA})(1000 \Omega) \\
& \mathrm{V}_{\min }=-4.99 \mathrm{~V}
\end{aligned}
$$

Calculate the voltage across the resistor for this minimum condition.
By Ohm's Law

$$
\mathrm{V}_{\mathrm{Rmin}}=\mathrm{I}_{\min } \mathrm{R}=(-0.01 \mathrm{~mA})(1000 \Omega)=-0.01 \mathrm{~V}
$$

## Diode Biasing Example

Consider the following circuit with a source voltage $\mathrm{V}_{\mathrm{S}}=5.0 \sin (10 \mathrm{t}) \mathrm{V}$, a load resistor $\mathrm{R}=1.00 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}$ and reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$.


Calculate the maximum voltage and current for forward bias, i.e. the diode is on.

Calculate the voltage across the resistor for this maximum condition.

Calculate the minimum voltage and current for reverse bias, i.e. the diode is off.

Calculate the voltage across the resistor for this minimum condition.

## Breakdown Diode Biasing Example

Consider the following diode circuit with a constant source voltage $\mathrm{V}_{\mathrm{s}}$, a load resistor $\mathrm{R}=1.00 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}$, reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$, and a breakdown voltage $\mathrm{V}_{\mathrm{br}}=26 \mathrm{~V}$.


Calculate the operating point if $V_{S}=-25 V$. Is the diode forward biased or reverse biased?

The diode is reverse biased
For reverse bias the diode can be in breakdown or not in breakdown.
Assume that the diode is not in breakdown, then

$$
\mathrm{I}=-\mathrm{I}_{0}=-0.01 \mathrm{~mA}
$$

By the load line equation (from KVL) the voltage is

$$
\mathrm{V}=\mathrm{V}_{\mathrm{S}}-\left(-\mathrm{I}_{0}\right) \mathrm{R}=-25.0-(-0.01 \mathrm{~mA})(1000 \Omega)=-24.99 \mathrm{~V}
$$

These values are possible
Note that if the diode is assumed to be in breakdown: $\mathrm{V}=-\mathrm{V}_{\mathrm{br}}=-26 \mathrm{~V}$ and
$\mathrm{I}=(1 / \mathrm{R})\left[\mathrm{V}_{\mathrm{S}}-\left(-\mathrm{V}_{\mathrm{br}}\right)\right]=(-25+26) / 1000=1 \mathrm{~mA}$
But, the current cannot be greater than - $\mathrm{I}_{0}$ in reverse bias.
The values are not possible

## Calculate the operating point if $V_{S}=-30 \mathrm{~V}$.

Note that the LL intercepts are $\mathrm{I}=0$ for $\mathrm{V}=\mathrm{V}_{\mathrm{S}}$ and $\mathrm{V}=0$ for $\mathrm{I}=\mathrm{V}_{\mathrm{S}} / \mathrm{R}$ Based on the $\mathrm{I}=0$ intercept, the diode should be in breakdown. Hence,

$$
\mathrm{V}=-\mathrm{V}_{\mathrm{br}}=-26 \mathrm{~V}
$$

By the load line equation (from KVL) the current is
$\mathrm{I}=(1 / \mathrm{R})\left[\mathrm{V}_{\mathrm{s}}-\left(-\mathrm{V}_{\mathrm{br}}\right)\right]=(1 / 1000)[-30-(-26)]=-4.0 \mathrm{~mA}$
Note that these values are possible.
(If the diode is incorrectly assumed to be not in breakdown, the result voltage value will be less than $-\mathrm{V}_{\mathrm{br}}$ which is not possible.)

## Breakdown Diode Biasing Example

Consider the following diode circuit with a constant source voltage $\mathrm{V}_{\mathrm{s}}$, a load resistor $\mathrm{R}=1.00 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}$, reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$, and a breakdown voltage $\mathrm{V}_{\mathrm{br}}=26 \mathrm{~V}$.


Calculate the operating point if $V_{S}=-25 \mathrm{~V}$. Is the diode forward biased or reverse biased?

Calculate the operating point if $V_{S}=-30 \mathrm{~V}$.

## Diode Biasing Example

Consider the following circuit with a constant source voltage $\mathrm{V}_{\mathrm{S}}$, an unknown load resistance $R$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.70 \mathrm{~V}$ and reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$.


Calculate the diode current for which the voltage is $V=-0.070 \mathrm{~V}$. Use the low-level-injection diode equation and assume room temperature.

The diode is reverse biased for a negative voltage.
The diode equation is $\mathrm{I}=\mathrm{I}_{0}[\exp (\mathrm{qV} / \mathrm{kT})-1]$
Then,

$$
\mathrm{I}=0.01 \mathrm{~mA}[\exp (-0.070 \mathrm{~V} / 0.0259 \mathrm{~V})-1]=-0.00933 \mathrm{~mA}
$$

Note that this current is $\mathrm{I}=-0.0933 \mathrm{I}_{0}$.
If the source voltage is $V_{S}=+4.0 \mathrm{~V}$, calculate the required resistance for a diode current of $I=+2.0 \mathrm{~mA}$.

The diode is forward biased when $\mathrm{V}_{\mathrm{S}}=+4.0 \mathrm{~V}$.
For forward bias (away from the knee of the IV curve), the diode voltage is

$$
\mathrm{V}=\mathrm{V}_{\mathrm{to}}=0.7 \mathrm{~V}
$$

The load line equation is

$$
-V_{S}+V+I R=0
$$

The required resistance is

$$
\mathrm{R}=(1 / \mathrm{I})\left[\mathrm{V}_{\mathrm{S}}-\left(\mathrm{V}_{\text {to }}\right)\right]=(1 / 0.002 \mathrm{~A})[4.0-0.7 \mathrm{~V}]=1.65 \mathrm{k} \Omega .
$$

If the source voltage is $V_{S}=+4.0 \mathrm{~V}$, calculate the required resistance for a diode current of $I=+4.0 \mathrm{~mA}$.

The required resistance is

$$
\mathrm{R}=(1 / \mathrm{I})\left[\mathrm{V}_{\mathrm{s}}-\left(\mathrm{V}_{\mathrm{to}}\right)\right]=(1 / 0.004 \mathrm{~A})[4.0-0.7 \mathrm{~V}]=0.825 \mathrm{k} \Omega .
$$

## Diode Biasing Example

Consider the following circuit with a constant source voltage $\mathrm{V}_{\mathrm{S}}$, an unknown load resistance R , and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.70 \mathrm{~V}$ and reverse saturation current of $\mathrm{I}_{0}=0.01 \mathrm{~mA}$.


Calculate the diode current for which the voltage is $V=-0.070 \mathrm{~V}$. Use the low-level-injection diode equation and assume room temperature.

If the source voltage is $V_{S}=+4.0 \mathrm{~V}$, calculate the required resistance for a diode current of $I=+2.0 \mathrm{~mA}$.

If the source voltage is $V_{S}=+4.0 \mathrm{~V}$, calculate the required resistance for a diode current of $I=+4.0 \mathrm{~mA}$.

## Diode Limiter Example

Consider the following limiting circuit with a source voltage $\mathrm{V}_{\mathrm{s}}$, resistances $\mathrm{R}_{\text {limiting }}=\mathrm{R}=1.0 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.70 \mathrm{~V}$, breakdown voltage $\mathrm{V}_{\mathrm{br}}=10 \mathrm{~V}$, and reverse saturation current of $\mathrm{I}_{0}=0.10 \mathrm{~mA}$.


If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, M i n}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the minimum input level.

The diode is reverse biased when $\mathrm{V}_{\mathrm{S}}=+10.0 \mathrm{~V}$.
For reverse bias (away from the knee of the IV curve), the diode may be in breakdown or not in breakdown. Consider a calculation of the diode voltage assuming that the diode did not have a breakdown point. If the calculated diode voltage is greater or equal to the breakdown voltage, then the diode is not in breakdown. If the calculated diode voltage is less than the breakdown voltage, the assumption is incorrect and the diode is in breakdown.
Assuming that the diode is not in breakdown, the diode current is $\mathrm{I}=-\mathrm{I}_{0}$.
The load voltage $\mathrm{V}_{0}$ can be calculated from the KCL equation

$$
\begin{aligned}
& \mathrm{V}_{0} / \mathrm{R}-\mathrm{I}+\left(\mathrm{V}_{0}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}_{\text {limiting }}=0 \\
& \mathrm{~V}_{0}=(1 / 2) \mathrm{R} \mathrm{I}+(1 / 2) \mathrm{V}_{\mathrm{S}}=(1 / 2)(1000)(-0.0001)+(1 / 2) 10=4.95 \mathrm{~V}
\end{aligned}
$$

Note that by KVL the diode voltage is $\mathrm{V}=-\mathrm{V}_{0}=-4.95 \mathrm{~V}>-\mathrm{V}_{\mathrm{br}}$.
The assumption was correct.
$V_{0}=+4.95 \mathrm{~V}$ (Diode is Reverse Biased with no Breakdown)
If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, \text { Min }}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the maximum input level.

The diode is reverse biased when $\mathrm{V}_{\mathrm{S}}=+30.0 \mathrm{~V}$.
Assuming that the diode is in breakdown, the diode voltage is $\mathrm{V}=-\mathrm{V}_{\mathrm{br}}$.
By KVL, the load voltage is $\mathrm{V}_{0}=-\mathrm{V}=-(-10 \mathrm{~V})$
The KCL equation gives

$$
\mathrm{I}=\mathrm{V}_{0} / \mathrm{R}+\left(\mathrm{V}_{0}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}_{\text {limiting }}=[2(10)-30] /(1000)=-10 \mathrm{~mA}
$$

Note that the diode current is $\mathrm{I}<-\mathrm{I}_{0}$, the assumption was correct.
$\mathrm{V}_{\mathbf{0}}=+10 \mathrm{~V}$ (Diode is Reverse Biased with Breakdown)

## Diode Limiter Example

Consider the following limiting circuit with a source voltage $\mathrm{V}_{\mathrm{s}}$, resistances $\mathrm{R}_{\text {limiting }}=\mathrm{R}=1.0 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.70 \mathrm{~V}$, breakdown voltage $\mathrm{V}_{\mathrm{br}}=10 \mathrm{~V}$, and reverse saturation current of $\mathrm{I}_{0}=0.10 \mathrm{~mA}$.


If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, M i n}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the minimum input level.

KVL gives $\mathrm{V}_{0}=-\mathrm{V}$ and KCL gives $\mathrm{V}_{0} / \mathrm{R}-\mathrm{I}+\left(\mathrm{V}_{0}-\mathrm{V}_{\mathrm{s}}\right) / \mathrm{R}_{\text {limiting }}=0$.
With $\mathrm{R}_{\text {limiting }}=\mathrm{R},-\mathrm{V}_{0} / \mathrm{R}-\mathrm{I}+\left(-\mathrm{V}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}=0$ or $\mathrm{V}=-(1 / 2) \mathrm{V}_{\mathrm{S}}-(1 / 2) \mathrm{RI}$.
For $\mathrm{V}_{\mathrm{S}, \mathrm{Min}}=+10.0 \mathrm{~V}$, the voltage intercept is $-(1 / 2) \mathrm{V}_{\mathrm{S}}=-5 \mathrm{~V}>-\mathrm{V}_{\mathrm{br}}$.
Hence, the KCL equation intersects the IV characteristic in the
reverse bias (no breakdown region).
Then, $\mathrm{I}=-\mathrm{I}_{0}$ and the KCL equation gives

$$
\mathrm{V}_{0}=(1 / 2) \mathrm{RI}+(1 / 2) \mathrm{V}_{\mathrm{S}}=(1 / 2)(1000)(-0.0001)+(1 / 2) 10=4.95 \mathrm{~V}
$$

$V_{0}=+4.95 \mathrm{~V}$ (Diode is Reverse Biased with no Breakdown)
If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, M i n}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the maximum input level.

KVL gives $\mathrm{V}_{0}=-\mathrm{V}$ and KCL gives $\mathrm{V}_{0} / \mathrm{R}-\mathrm{I}+\left(\mathrm{V}_{0}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}_{\text {limiting }}=0$.
With $\mathrm{R}_{\text {limiting }}=\mathrm{R},-\mathrm{V}_{0} / \mathrm{R}-\mathrm{I}+\left(-\mathrm{V}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}=0$ or $\mathrm{V}=-(1 / 2) \mathrm{V}_{\mathrm{S}}-(1 / 2) \mathrm{RI}$.
For $\mathrm{V}_{\mathrm{S}, \mathrm{Max}}=+30.0 \mathrm{~V}$, the voltage intercept is $-(1 / 2) \mathrm{V}_{\mathrm{S}}=-15 \mathrm{~V}<-\mathrm{V}_{\mathrm{br}}$.
Hence, the KCL equation intersects the IV characteristic in the reverse bias with breakdown region.
Then, $\mathrm{V}_{0}=-\mathrm{V}=-\left(-\mathrm{V}_{\mathrm{br}}\right)=15 \mathrm{~V}$ and the KCL equation gives

$$
\mathrm{I}=\mathrm{V}_{0} / \mathrm{R}+\left(\mathrm{V}_{0}-\mathrm{V}_{\mathrm{S}}\right) / \mathrm{R}_{\text {limiting }}=[2(10)-30] /(1000)=-10 \mathrm{~mA}
$$

## $\mathrm{V}_{0}=+10 \mathrm{~V}$ (Diode is Reverse Biased with Breakdown)

## Diode Limiter Example

Consider the following limiting circuit with a source voltage $\mathrm{V}_{\mathrm{s}}$, resistances $\mathrm{R}_{\text {limiting }}=\mathrm{R}=1.0 \mathrm{k} \Omega$, and diode IV characteristics of turn-on voltage $\mathrm{V}_{\mathrm{to}}=0.70 \mathrm{~V}$, breakdown voltage $\mathrm{V}_{\mathrm{br}}=10 \mathrm{~V}$, and reverse saturation current of $\mathrm{I}_{0}=0.10 \mathrm{~mA}$.


If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, M i n}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the minimum input level.

If the source voltage is a square wave that varies between $V_{S, M a x}=+30.0 \mathrm{~V}$ and $V_{S, M i n}=+10.0 \mathrm{~V}$, calculate the load voltage $V_{0}$ for the maximum input level.

# EE 2200 INTRO. TO ELECTRONIC DEVICES 

## ECE, Missouri S\&T

© 2023 Steve E. Watkins

# WORKSHEETS <br> FOR <br> EXAMINATION 2 

Topics Include:
Bipolar Junction Transistor (BJT) Physics
BJT Circuits
Field Effect Transistor (FET) Physics
JFET, Depletion-Mode MOSFET, and Enhancement Mode MOSFET Devices
JFET and MOSFET Circuits

Boltzmann's constant:
Planck's constant
Electronic charge:
kT at 300 K
Free-space permittivity
Relative permittivity
Bandgap energies
$\mathrm{k}=1.381 \times 10^{-23} \mathrm{~J} / \mathrm{K}=8.618 \times 10^{-5} \mathrm{eV} / \mathrm{K}$
$\mathrm{h}=4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}=6.626 \times 10^{-34} \mathrm{~J}-\mathrm{sec}$
$\mathrm{q}=1.602 \times 10^{-19} \mathrm{C}$
$\mathrm{kT}=0.0259 \mathrm{eV} \quad$ eV-J conversion $\quad 1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$
$\varepsilon_{\mathrm{O}}=8.854 \times 10^{-14} \mathrm{~F} / \mathrm{cm} \quad$ Speed of Light $\quad \mathrm{c}=2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s}$
Si: $11.9 \quad$ Ge: 16.0
Si: $1.12 \mathrm{eV} \quad$ Ge: 0.67 eV

GaAs: 13.1
GaAs: 1.42 eV

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{vs}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{V}_{\mathrm{EB}}=\mathrm{V}_{\mathrm{to}}$ ):

$$
-\mathrm{V}_{\mathrm{BB}}+\mathrm{i}_{\mathrm{B}} \mathrm{R}_{\mathrm{b}}+\mathrm{V}_{\mathrm{to}}=0 \text { or } \mathrm{i}_{\mathrm{B}}=\left(1 / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right)
$$

and

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\beta / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) \\
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=(200 / 10,000)(2.7-0.7)=0.040 \mathrm{~A}=40 \mathrm{~mA}
\end{aligned}
$$

Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{V}_{\mathrm{EC}}=0$ or $\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}$
$\mathrm{V}_{\mathrm{EC}}=16-(0.040)(200)=8.0 \mathrm{~V}$

## Check KCL for the transistor.

The currents are:

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=(200 / 10,000)(2.7-0.7)=0.040 \mathrm{~A}=40 \mathrm{~mA} \\
& \mathrm{i}_{\mathrm{B}}=(1 / 10,000)(2.7-0.7)=0.00020 \mathrm{~A}=0.20 \mathrm{~mA} \\
& \mathrm{i}_{\mathrm{E}}=\mathrm{i}_{\mathrm{C}} / \alpha_{0}=(0.040) /[200 /(1+200)]=0.0402 \mathrm{~A}=40.2 \mathrm{~mA}
\end{aligned}
$$

Kirchhoff's Current Law for the transistor gives

$$
\begin{aligned}
& +\mathrm{i}_{\mathrm{C}}+\mathrm{i}_{\mathrm{B}}-\mathrm{i}_{\mathrm{E}}=0 \\
& 40 \mathrm{~mA}+0.2 \mathrm{~mA}-40.2 \mathrm{~mA}=0
\end{aligned}
$$

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{vs}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.

Check KCL for the transistor.

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=4.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V and assume that the saturation voltage is $\mathrm{v}_{\mathrm{EC}, \mathrm{SAT}}=0.2 \mathrm{~V}$. Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $\nu_{E C}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{V}_{\mathrm{EB}}=\mathrm{V}_{\mathrm{to}}$ ):

$$
-\mathrm{V}_{\mathrm{BB}}+\mathrm{i}_{\mathrm{B}} \mathrm{R}_{\mathrm{b}}+\mathrm{V}_{\mathrm{to}}=0 \text { or } \mathrm{i}_{\mathrm{B}}=\left(1 / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right)
$$

and

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\beta / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) \\
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=(200 / 10,000)(4.7-0.7)=0.080 \mathrm{~A}=80 \mathrm{~mA}
\end{aligned}
$$

Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{V}_{\mathrm{EC}}=0$ or $\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}$

$$
\mathrm{v}_{\mathrm{EC}}=16-(0.080)(200)=0 \mathrm{~V} \text { ???????? }
$$

Since $\mathrm{vec}^{<}<\mathrm{V}_{\mathrm{EC}, \mathrm{SAT}}=0.2 \mathrm{~V}$, the assumption of operation in the active region is invalid, e.g. ic $\neq \beta \mathrm{i}_{\mathrm{B}}$. For operation in the saturation region, use the approximation $\mathrm{V}_{\mathrm{EC}} \sim \mathrm{V}_{\mathrm{EC}, \mathrm{SAT}}=0.2 \mathrm{~V}$, Then,

$$
\begin{aligned}
& -\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{V}_{\mathrm{EC}, \mathrm{SAT}}=0 \text { or } \mathrm{i}_{\mathrm{C}}=\left(1 / \mathrm{R}_{\mathrm{c}}\right)\left(\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\mathrm{EC}, \mathrm{SAT}}\right) \\
& \mathrm{i}_{\mathrm{C}}=(1 / 200)(16-0.2)=0.079 \mathrm{~A}=79 \mathrm{~mA}
\end{aligned}
$$

Note that $i_{B}$ is unchanged with

$$
\mathrm{i}_{\mathrm{B}}=\left(1 / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right)=(1 / 10,000)(4.7-0.7)=0.00040 \mathrm{~A}=0.40 \mathrm{~mA}
$$ and the effective gain is $\quad \beta_{\text {effective }}=i_{C} / i_{B}=79 / 0.04=197.5 \neq \beta=200$

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=4.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V and assume that the saturation voltage is $\mathrm{V}_{\mathrm{EC}, \mathrm{SAT}}=0.2 \mathrm{~V}$. Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $R_{e}=100 \Omega$, and $R_{c}=100 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{V}_{\mathrm{EB}}=\mathrm{V}_{\mathrm{to}}$ ):
$-V_{B B}+i_{B} R_{b}+i_{E} R_{e}+V_{\text {to }}=0$
Since $\beta / \alpha_{o}=1+\beta$ and $i_{C}=\alpha_{o} i_{E}=\beta i_{B}$, then $i_{E}=(1+\beta) i_{B}$ and $\mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{R}_{\mathrm{e}}(1+\beta)+\mathrm{R}_{\mathrm{b}}\right]$
Also,

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{Re}_{\mathrm{e}}(1+\beta) /(\beta)+\mathrm{R}_{\mathrm{b}} /(\beta)\right] \\
& \mathrm{i}_{\mathrm{C}}=(2.7-0.7) /[100(201 / 200)+10,000 / 200]=0.0133 \mathrm{~A}
\end{aligned}
$$

Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{V}_{\mathrm{EC}}=0$ or $\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}-\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}$
$\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}}\left(\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{e}} / \alpha_{\mathrm{o}}\right)=16-0.0133[100+100(201) /(200)]=13.3 \mathrm{~V}$

Calculate the current $i_{C}$ if the gain changes to 150.
As before

$$
\mathrm{i}_{\mathrm{C}}=(2.7-0.7) /[100(151 / 150)+10,000 / 150]=0.0120 \mathrm{~A}
$$

Note that the current changes by $10 \%$.
The condition $R_{e} \gg R_{b} /(\beta)$ was not satisfied.

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{e}}=100 \Omega \\
& \mathrm{R}_{\mathrm{b}} / \beta=10,000 / 200=50 \Omega
\end{aligned}
$$

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $R_{e}=100 \Omega$, and $R_{c}=100 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.

Calculate the current ic if the gain changes to 150.

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $R_{e}=\mathbf{5 0 0} \Omega$, and $R_{c}=\mathbf{5 0 0} \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $\nu_{E C}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{V}_{\mathrm{EB}}=\mathrm{V}_{\mathrm{to}}$ ):
$-V_{B B}+i_{B} R_{b}+i_{E} R_{e}+V_{\text {to }}=0$
Since $\beta / \alpha_{o}=1+\beta$ and $i_{C}=\alpha_{o} i_{E}=\beta i_{B}$, then $i_{E}=(1+\beta) i_{B}$ and $\mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{Re}_{\mathrm{e}}(1+\beta)+\mathrm{R}_{\mathrm{b}}\right]$ or
Also,

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{te})}\right) /\left[\mathrm{Re}(1+\beta) /(\beta)+\mathrm{R}_{\mathrm{b}} /(\beta)\right] \\
& \mathrm{i}_{\mathrm{C}}=(2.7-0.7) /[500(201 / 200)+10,000 / 200]=0.00362 \mathrm{~A}
\end{aligned}
$$

Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{V}_{\mathrm{EC}}=0$ or $\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}-\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}$
$\mathrm{V}_{\mathrm{EC}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}}\left(\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{e}} / \alpha_{o}\right)=16-0.00362[500+500(201) /(200)]=12.4 \mathrm{~V}$

Calculate the current $i_{C}$ if the gain changes to 150.
As before

$$
\mathrm{i}_{\mathrm{C}}=(2.7-0.7) /[500(151 / 150)+10,000 / 150]=0.00351 \mathrm{~A}
$$

Note that the current changes by about $3 \%$.
Note the condition $R_{e}>R_{b} /(\beta)$ by a factor of ten.

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{e}}=500 \Omega \\
& \mathrm{R}_{\mathrm{b}} / \beta=10,000 / 200=50 \Omega
\end{aligned}
$$

## Common Emitter Circuit Example

Consider a pnp BJT circuit with $\beta=200, \mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $\mathrm{R}_{\mathrm{e}}=\mathbf{5 0 0} \Omega$, and $\mathrm{R}_{\mathrm{c}}=\mathbf{5 0 0} \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the operating point $i_{C}$ and $v_{E C}$.

Calculate the current ic if the gain changes to 150.

## Darlington Amplifier Circuit Example

Consider an npn BJT circuit with $\beta=50, \mathrm{R}_{\mathrm{b}}=100.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=3.4 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{vs}^{=}=0$.


Calculate the currents $i_{C 1}$ and $i_{C 2}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{v}_{\mathrm{BE} 1}=\mathrm{v}_{\mathrm{BE} 2}=\mathrm{V}_{\mathrm{to}}$ ):

$$
-\mathrm{V}_{\mathrm{BB}}+\mathrm{i}_{\mathrm{B} 1} \mathrm{R}_{\mathrm{b}}+2 \mathrm{~V}_{\mathrm{to}}=0 \text { or } \mathrm{i}_{\mathrm{B} 1}=\left(1 / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-2 \mathrm{~V}_{\mathrm{to}}\right)
$$

and

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{C} 1}=\beta \mathrm{i}_{\mathrm{B} 1}=\left(\beta / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-2 \mathrm{~V}_{\mathrm{to}}\right) \\
& \mathrm{i}_{\mathrm{C} 1}=\beta \mathrm{i}_{\mathrm{B} 1}=(50 / 100,000)(3.4-1.4)=0.001 \mathrm{~A}=1.0 \mathrm{~mA} \\
& \mathrm{i}_{\mathrm{C} 2}=\beta_{2} \mathrm{i}_{\mathrm{B} 2}=\beta_{2} \mathrm{i}_{\mathrm{E} 1}=\beta_{2} \mathrm{i}_{\mathrm{C} 1} / \alpha_{\mathrm{o} 1}=\beta_{2} \beta_{1} \mathrm{i}_{\mathrm{B} 1} / \alpha_{\mathrm{ol}}=\beta_{2}\left(1+\beta_{1}\right) \mathrm{i}_{\mathrm{B} 1} \\
& \mathrm{i}_{\mathrm{C} 2}=\beta(1+\beta)\left(1 / \mathrm{R}_{\mathrm{b}}\right)\left(\mathrm{V}_{\mathrm{BB}}-2 \mathrm{~V}_{\mathrm{to}}\right) \\
& \mathrm{i}_{\mathrm{C} 2}=50(1+50)(1 / 100,000)(3.4-1.4)=0.051 \mathrm{~A}=51 \mathrm{~mA} \\
& \\
& \mathrm{i}_{\mathrm{C} \text { Total }}=\mathrm{i}_{\mathrm{C} 1}+\mathrm{i}_{\mathrm{C} 2}=52 \mathrm{~mA}
\end{aligned}
$$

Calculate the voltages $v_{C E 1}$ and $v_{C E 2}$.
Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):

$$
\begin{aligned}
& -\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{CTotal}} \mathrm{R}_{\mathrm{c}}+\mathrm{v}_{\mathrm{CE} 1}+\mathrm{V}_{\text {to }}=0 \quad \text { or } \mathrm{V}_{\mathrm{CE} 1}=\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\text {to }}-\mathrm{i}_{\mathrm{CTotal}} \mathrm{R}_{\mathrm{c}} \\
& \mathrm{v}_{\mathrm{CE} 1}=\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\text {to }}-\mathrm{i}_{\mathrm{CTota1}} \mathrm{R}_{\mathrm{c}}=16-0.7-0.052(200)=4.9 \mathrm{~V} \\
& -\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{CTotal}} \mathrm{R}_{\mathrm{c}}+\mathrm{v}_{\mathrm{CE} 2}=0 \text { or } \mathrm{V}_{\mathrm{CE} 2}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{CTotal}} \mathrm{R}_{\mathrm{c}} \\
& \mathrm{~V}_{\mathrm{CE} 2}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{CTotal}} \mathrm{R}_{\mathrm{c}}=16-0.052(200)=5.6 \mathrm{~V}
\end{aligned}
$$

## Darlington Amplifier Circuit Example

Consider an npn BJT circuit with $\beta=50, \mathrm{R}_{\mathrm{b}}=100.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=3.4 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, and $R_{c}=200 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V . Let $\mathrm{v}_{\mathrm{S}}=0$.


Calculate the currents $i_{C 1}$ and $i_{C 2}$.

Calculate the voltages $v_{\text {CEI }}$ and $v_{\text {CE2 }}$.

## Common Emitter Circuit Example

Consider an npn BJT circuit with $\beta=200$, $\mathrm{R}_{1}=40.0 \mathrm{k} \Omega, \mathrm{R}_{2}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{CC}}=15 \mathrm{~V}$, $\mathrm{R}_{\mathrm{e}}=1.0 \mathrm{k} \Omega$, and $\mathrm{R}_{\mathrm{c}}=1.0 \mathrm{k} \Omega$. Assume the base-emitter turn-on voltage is 0.7 V .

Calculate the Thevenin equivalent with
 respect to the Base circuit, i.e. $V_{B B}$ and $R_{b}$.

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{BB}}=\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{CC}}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=15 \mathrm{~V}[10,000 /(10,000+40,000)] \\
& \mathrm{V}_{\mathrm{BB}}=3.0 \mathrm{~V} \\
& \mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\mathrm{TH}}=\mathrm{R}_{1} \| \mathrm{R}_{2}=\left[\mathrm{R}_{1} \mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=[(10,000)(40,000) /(10,000+40,000)] \\
& \mathrm{R}_{\mathrm{b}}=8,000 \Omega
\end{aligned}
$$

Calculate the operating point $i_{C}$ and $v_{C E}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{v}_{\mathrm{BE}}=\mathrm{V}_{\mathrm{to}}$ ):

$$
-\mathrm{V}_{\mathrm{BB}}+\mathrm{i}_{\mathrm{B}} \mathrm{R}_{\mathrm{b}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{V}_{\mathrm{to}}=0
$$

Since $\beta / \alpha_{o}=1+\beta$ and $i_{C}=\alpha_{o} i_{E}=\beta i_{B}$, then $i_{E}=(1+\beta) i_{B}$ and

$$
\mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{Re}_{\mathrm{e}}(1+\beta)+\mathrm{R}_{\mathrm{b}}\right]
$$

$$
\mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{Re}_{\mathrm{e}}(1+\beta) /(\beta)+\mathrm{R}_{\mathrm{b}} /(\beta)\right]
$$

$$
\mathrm{i}_{\mathrm{C}}=(3.0-0.7) /[1000(201 / 200)+8,000 / 200]=0.00220 \mathrm{~A}
$$

Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{v}_{\mathrm{CE}}=0$ or $\mathrm{v}_{\mathrm{CE}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}-\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}$
$\mathrm{V}_{\mathrm{CE}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}}\left(\mathrm{R}_{\mathrm{c}}+\mathrm{Re}_{\mathrm{e}} / \alpha_{\mathrm{o}}\right)=15-0.00220[1000+1000(201) /(200)]=10.6 \mathrm{~V}$

## Common Emitter Circuit Example

Consider an npn BJT circuit with $\beta=200$, $\mathrm{R}_{1}=40.0 \mathrm{k} \Omega, \mathrm{R}_{2}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{CC}}=15 \mathrm{~V}$, $\mathrm{R}_{\mathrm{e}}=1.0 \mathrm{k} \Omega$, and $\mathrm{R}_{\mathrm{c}}=1.0 \mathrm{k} \Omega$. Assume the base-emitter turn-on voltage is 0.7 V .

Calculate the Thevenin equivalent with
 respect to the Base circuit, i.e. $V_{B B}$ and $R_{b}$.

Calculate the operating point ic and $v_{C E}$.

## Common Emitter Circuit Example

Consider an npn BJT circuit with $\beta=200$, $\mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $R_{e}=500 \Omega$, and $R_{c}=500 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V .

Calculate the value of resistors $R_{1}$ and $R_{2}$ that will produce the equivalent circuit for $R_{b}=10.0 \mathrm{k} \Omega$ and $V_{B B}=2.7 \mathrm{~V}$.


The Thevenin equivalent for the circuit to the left of the Base terminal

$$
\mathrm{V}_{\mathrm{BB}}=\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{CC}}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right] \quad \mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\mathrm{TH}}=\mathrm{R}_{1} \| \mathrm{R}_{2}=\left[\mathrm{R}_{1} \mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]
$$

Hence,
$\mathrm{V}_{\mathrm{BB}}=\mathrm{V}_{\mathrm{CC}}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=\mathrm{V}_{\mathrm{CC}}\left[1 / \mathrm{R}_{1}\right]\left[\mathrm{R}_{1} \mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=\mathrm{V}_{\mathrm{CC}}\left[1 / \mathrm{R}_{1}\right] \mathrm{R}_{\mathrm{b}}$ Similarly,
$\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\mathrm{BB}}=\mathrm{V}_{\mathrm{CC}}\left[\mathrm{R}_{1} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=\mathrm{V}_{\mathrm{CC}}\left[1 / \mathrm{R}_{2}\right]\left[\mathrm{R}_{1} \mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=\mathrm{V}_{\mathrm{CC}}\left[1 / \mathrm{R}_{2}\right] \mathrm{R}_{\mathrm{b}}$
Then,
$\mathrm{R}_{1}=\left(\mathrm{V}_{\mathrm{CC}} / \mathrm{V}_{\mathrm{BB}}\right) \mathrm{R}_{\mathrm{b}}=(16 \mathrm{~V} / 2.7 \mathrm{~V}) 10.0 \mathrm{k} \Omega=59.26 \mathrm{k} \Omega$
$\mathrm{R}_{2}=\left[\mathrm{V}_{\mathrm{CC}} /\left(\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\mathrm{BB}}\right)\right] \mathrm{R}_{\mathrm{b}}=[16 \mathrm{~V} /(16-2.7 \mathrm{~V})] 10.0 \mathrm{k} \Omega=12.03 \mathrm{k} \Omega$
Calculate the operating point $i_{C}$ and $v_{C E}$.
Kirchhoff's-Voltage-Law on Base Side ( $\mathrm{v}_{\mathrm{BE}}=\mathrm{V}_{\mathrm{to}}$ ):

$$
-\mathrm{V}_{\mathrm{BB}}+\mathrm{i}_{\mathrm{B}} \mathrm{R}_{\mathrm{b}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{V}_{\mathrm{to}}=0
$$

Since $\beta / \alpha_{o}=1+\beta$ and $i_{C}=\alpha_{o} i_{E}=\beta i_{B}$, then $i_{E}=(1+\beta) i_{B}$ and
$\mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\mathrm{R}_{\mathrm{e}}(1+\beta)+\mathrm{R}_{\mathrm{b}}\right]$
$\mathrm{i}_{\mathrm{C}}=\beta \mathrm{i}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{BB}}-\mathrm{V}_{\mathrm{to}}\right) /\left[\operatorname{Re}(1+\beta) /(\beta)+\mathrm{R}_{\mathrm{b}} /(\beta)\right]$
$\mathrm{i}_{\mathrm{C}}=(2.7-0.7) /[500(201 / 200)+10,000 / 200]=0.00362 \mathrm{~A}$
Kirchhoff's-Voltage-Law on Collector Side (the Load-Line Equation):
$-\mathrm{V}_{\mathrm{CC}}+\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{c}}+\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}+\mathrm{v}_{\mathrm{CE}}=0$ or $\mathrm{v}_{\mathrm{CE}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{i}} \mathrm{R}_{\mathrm{c}}-\mathrm{i}_{\mathrm{E}} \mathrm{R}_{\mathrm{e}}$
$\mathrm{V}_{\mathrm{CE}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{i}_{\mathrm{C}}\left(\mathrm{R}_{\mathrm{c}}+\mathrm{R}_{\mathrm{e}} / \alpha_{\mathrm{o}}\right)=16-0.00362[500+500(201) /(200)]=12.4 \mathrm{~V}$

## Common Emitter Circuit Example

Consider an npn BJT circuit with $\beta=200$, $\mathrm{R}_{\mathrm{b}}=10.0 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{BB}}=2.7 \mathrm{~V}, \mathrm{~V}_{\mathrm{CC}}=16 \mathrm{~V}$, $\mathrm{R}_{\mathrm{e}}=500 \Omega$, and $\mathrm{R}_{\mathrm{c}}=500 \Omega$. Assume the base-emitter turn-on voltage is 0.7 V .

Calculate the value of resistors $R_{1}$ and $R_{2}$ that will produce the equivalent circuit for $R_{b}=10.0 \mathrm{k} \Omega$ and $V_{B B}=2.7 \mathrm{~V}$.


Calculate the operating point $i_{C}$ and $v_{C E}$.

## Common Source JFET Circuit Example

Consider a n-channel JFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}, \mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{R}_{\mathrm{d}}=? ? \Omega$, $\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\right.$ since $\left.\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.

Calculate the maximum input voltage $V_{i}$ for which the JFET operating point has $i_{D S}=0$ A. Also, calculate $v_{D S}$.


Since $\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{GS}}$, the maximum input voltage or maximum $\mathrm{v}_{\mathrm{GS}}$ for $\mathrm{i}_{\mathrm{DS}}=0$ occurs at $\mathrm{V}_{\mathrm{GS}}=-\mathrm{V}_{\mathrm{po}}=-5.0 \mathrm{~V}$ as seen in equation $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left(1+\mathrm{V}_{\mathrm{GS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}$. The load line gives $\mathrm{V}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{d}}=15 \mathrm{~V}-0=15 \mathrm{~V}$.

Hence, the input voltage $\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=-5.0 \mathrm{~V}$ gives an operating point of $v_{D S}=15 \mathrm{~V}$ and $\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}$.

Calculate the input voltage $V_{i}$ for which the JFET operating point has $i_{D S}=I_{D S S}$ and the JFET is just in saturation. Also, calculate vDs.

Note that $i_{D S}=I_{D S S}$ only for saturation with the input voltage $v_{G S}=0$. Saturation just occurs for $\mathrm{v}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{po}}=5 \mathrm{~V}$.

Hence, the input voltage $\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=0 \mathrm{~V}$ gives an operating point of $\mathrm{v}_{\mathrm{DS}}=5 \mathrm{~V}$ and $\mathrm{i}_{\mathrm{DS}}=1 \mathrm{~mA}$.

Calculate the resistance $R_{d}$ for which the JFET circuit load line passes through both of these operating points.

The load line is $\mathrm{v}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{d}}$. One intercept corresponds to the first operating point $\mathrm{v}_{\mathrm{DS}}=15 \mathrm{~V}$ and $\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}$. The second operating point $\mathrm{v}_{\mathrm{DS}}=5 \mathrm{~V}$ and $\mathrm{i}_{\mathrm{DS}}=1 \mathrm{~mA}$ requires
$\mathrm{R}_{\mathrm{d}}=\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{DS}}\right) / \mathrm{i}_{\mathrm{DS}}=(15-5) / 0.001=10 \mathrm{k} \Omega$.

## Common Source JFET Circuit Example

Consider a n-channel JFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}, \mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{R}_{\mathrm{d}}=? ? \Omega$, $\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\right.$ since $\left.\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.

Calculate the maximum input voltage $V_{i}$ for which the JFET operating point has $i_{D S}=0$ A. Also, calculate $v_{D S}$.


Calculate the input voltage $V_{i}$ for which the JFET operating point has $i_{D S}=I_{D S S}$ and the JFET is just in saturation. Also, calculate $v_{D S}$.

Calculate the resistance $R_{d}$ for which the JFET circuit load line passes through both of these operating points.

## Common Source MOSFET Circuit Examples

Consider a n-channel depletion-mode
MOSFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}$, $\mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{~V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.

Calculate the input voltage $V_{i}$ for which the MOSFET circuit load line has $i_{D S}=0 \mathrm{~A}$, $i_{D S}=1 \mathrm{~mA}$, and $i_{D S}=2 \mathrm{~mA}$ for saturation.


Note that for saturation ids $=\mathrm{I}_{\mathrm{DSS}}\left(1+\mathrm{V}_{\mathrm{GS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}$.
Hence, for $\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}, \mathrm{~V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS}}=-\mathrm{V}_{\mathrm{po}}=-5.0 \mathrm{~V}$
Hence for $\mathrm{i}_{\mathrm{DS}}=1 \mathrm{~mA}$ for $\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}$
Hence, for $\mathrm{i}_{\mathrm{DS}}=2 \mathrm{~mA}, \mathrm{~V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS}}=+\mathrm{V}_{\mathrm{po}}(\sqrt{2}-1)=+2.07 \mathrm{~V}$
Calculate the resistance $R_{d}$ for which the MOSFET circuit load line passes through operating points $\left(v_{D S}=15 \mathrm{~V}, i_{D S}=0 \mathrm{~A}\right)$ and $\left(v_{D S}=5 \mathrm{~V}, i_{D S}=1 \mathrm{~mA}\right)$.

The load line is $v_{D S}=V_{D D}-i_{D S} R_{d}$. One intercept is $\left(v_{D S}=15 \mathrm{~V}\right.$, $\left.\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}\right)$.
The second operating point $\mathrm{v}_{\mathrm{DS}}=5 \mathrm{~V}$ and $\mathrm{i}_{\mathrm{DS}}=1 \mathrm{~mA}$ requires
$\mathrm{R}_{\mathrm{d}}=\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{DS}}\right) / \mathrm{i}_{\mathrm{DS}}=(15-5) / 0.001=10 \mathrm{k} \Omega$.

Consider a n-channel enhancement-mode MOSFET circuit with $\mathrm{V}_{\text {on }}=2.0 \mathrm{~V}$, $\mathrm{K}=0.04 \mathrm{~mA} / \mathrm{V}^{2}, \mathrm{VGS}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.

Calculate the input voltage $V_{i}$ for which the MOSFET circuit load line has $i_{D S}=0 \mathrm{~A}$,
 $i_{D S}=1 \mathrm{~mA}$, and $i_{D S}=2 \mathrm{~mA}$ for saturation.

Note that for saturation $\mathrm{i}_{\mathrm{DS}}=K \mathrm{~V}_{\mathrm{on}}{ }^{2}\left(\mathrm{~V}_{\mathrm{GS}} / \mathrm{V}_{\text {on }}-1\right)^{2}$.
Hence, for $\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}, \mathrm{~V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\text {on }}=2.0 \mathrm{~V}$
Hence for $\mathrm{i}_{\mathrm{DS}}=1 \mathrm{~mA}$ for $\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS}}=07.0 \mathrm{~V}$
Hence, for $\mathrm{i}_{\mathrm{DS}}=2 \mathrm{~mA}, \mathrm{~V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}=\mathrm{VGS}=+\mathrm{V}_{\mathrm{on}}(4.54)=+9.07 \mathrm{~V}$
Calculate the resistance $R_{d}$ for which the MOSFET circuit load line passes through operating points $\left(v_{D S}=15 \mathrm{~V}, i_{D S}=0 \mathrm{~A}\right)$ and $\left(v_{D S}=5 \mathrm{~V}, i_{D S}=1 \mathrm{~mA}\right)$.

The load line is the same, i.e. $\mathrm{v}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{d}}$. Then
$\mathrm{R}_{\mathrm{d}}=\left(\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{DS}}\right) / \mathrm{i}_{\mathrm{DS}}=(15-5) / 0.001=10 \mathrm{k} \Omega$.

## Common Source MOSFET Circuit Examples

Consider a n-channel depletion-mode MOSFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}$, $\mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{~V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$

Calculate the input voltage $V_{i}$ for which the MOSFET circuit load line has $i_{D S}=0 \mathrm{~A}$,
 $i_{D S}=1 \mathrm{~mA}$, and $i_{D S}=2 \mathrm{~mA}$ for saturation .

Calculate the resistance $R_{d}$ for which the MOSFET circuit load line passes through operating points $\left(v_{D S}=15 \mathrm{~V}, i_{D S}=0 \mathrm{~A}\right)$ and $\left(v_{D S}=5 \mathrm{~V}, i_{D S}=1 \mathrm{~mA}\right)$.

Consider a n-channel enhancement-mode
MOSFET circuit with $\mathrm{V}_{\mathrm{on}}=2.0 \mathrm{~V}$,
$\mathrm{K}=0.04 \mathrm{~mA} / \mathrm{V}^{2}, \mathrm{~V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}=\mathrm{V}_{\mathrm{i}}\left(\mathrm{i}_{\mathrm{G}}=0\right)$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.

Calculate the input voltage $V_{i}$ for which the MOSFET circuit load line has $i_{D S}=0 \mathrm{~A}$,
 $i_{D S}=1 \mathrm{~mA}$, and $i_{D S}=2 \mathrm{~mA}$ for saturation.

Calculate the resistance $R_{d}$ for which the MOSFET circuit load line passes through operating points $\left(v_{D S}=15 \mathrm{~V}, i_{D S}=0 \mathrm{~A}\right)$ and $\left(v_{D S}=5 \mathrm{~V}, i_{D S}=1 \mathrm{~mA}\right)$.

## MOSFET Current Example

An enhancement-mode MOSFET (n-channel) has

$$
\begin{aligned}
& \mathrm{V}_{\text {on }}=0.50 \mathrm{~V} \text { and } \\
& \mathrm{K}=0.20 \mathrm{~mA} / \mathrm{V}^{2},
\end{aligned}
$$

## Saturation Conditions

$\mathrm{v}_{\mathrm{DS}}-\mathrm{v}_{\mathrm{GS}} \geq-\mathrm{V}_{\text {on }}$
MOSFET Relationships

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{DS}}=\mathrm{KV}_{\mathrm{on}}{ }^{2}\left[2\left(\frac{\mathrm{v}_{\mathrm{GS}}}{v_{\text {on }}}-1\right)\left(\frac{\mathrm{v}_{\mathrm{DS}}}{v_{\text {on }}}\right)-\left(\frac{\mathrm{v}_{\mathrm{DS}}}{v_{\text {on }}}\right)^{2}\right] \\
& \mathrm{i}_{\mathrm{DS}}=\mathrm{KV}_{\mathrm{on}}{ }^{2}\left(\frac{\mathrm{v}_{\mathrm{GS}}}{v_{\text {on }}}-1\right)^{2}
\end{aligned}
$$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=0 \mathrm{~V}$
Since $\mathrm{VGS}_{\mathrm{GS}}=0<\mathrm{V}_{\mathrm{on}}=0.50 \mathrm{~V}$, the current is zero.
Then, $\mathrm{i}_{\mathrm{DS}}=0$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=+3.0 \mathrm{~V}$
Since $\mathrm{v}_{\mathrm{DS}}-\mathrm{V}_{\mathrm{GS}}=5-3=2 \mathrm{~V} \geq-\mathrm{V}_{\text {on }}=-0.5 \mathrm{~V}$ (inequality satisfied), the MOSFET has saturated operation.

Then, $\mathrm{i}_{\mathrm{DS}}=\mathrm{K} \mathrm{V}_{\mathrm{on}}{ }^{2}\left[\left(\mathrm{VGS} / \mathrm{V}_{\mathrm{on}}\right)-1\right]^{2}=(0.20)(0.50)^{2}[(3.0 / 0.5)-1]^{2}=1.25 \mathrm{~mA}$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=+6.0 \mathrm{~V}$
Since $\mathrm{V}_{\mathrm{DS}}-\mathrm{VGS}_{\mathrm{GS}}=5-6=-1 \mathrm{~V}<-\mathrm{V}_{\text {on }}=-0.5 \mathrm{~V}$, (inequality not satisfied) the MOSFET has unsaturated operation.

Then, $i_{\mathrm{DS}}=\mathrm{K} \mathrm{V}_{\mathrm{on}}{ }^{2}\left\{2\left[\left(\mathrm{vgS} / \mathrm{V}_{\mathrm{on}}\right)-1\right]\left(\mathrm{vdS} / \mathrm{V}_{\mathrm{on}}\right)-\left(\mathrm{vds} / \mathrm{V}_{\mathrm{on}}\right)^{2}\right\}$

$$
\mathrm{i}_{\mathrm{DS}}=(0.20)(0.50)^{2}\left\{2[(6.0 / 0.5)-1](5.0 / 0.5)-(5.0 / 0.5)^{2}\right\}=6.0 \mathrm{~mA}
$$

## MOSFET Current Example

An enhancement-mode MOSFET (n-channel) has

$$
\begin{aligned}
& \mathrm{V}_{\text {on }}=0.50 \mathrm{~V} \text { and } \\
& \mathrm{K}=0.20 \mathrm{~mA} / \mathrm{V}^{2},
\end{aligned}
$$

## Saturation Conditions

$\mathrm{v}_{\mathrm{DS}}-\mathrm{v}_{\mathrm{GS}} \geq-\mathrm{V}_{\text {on }}$
MOSFET Relationships

$$
\begin{aligned}
& i_{D S}=K V_{o n}^{2}\left[2\left(\frac{v_{G S}}{v_{o n}}-1\right)\left(\frac{v_{D S}}{v_{o n}}\right)-\left(\frac{v_{D S}}{v_{o n}}\right)^{2}\right] \\
& i_{D S}=K V_{o n}^{2}\left(\frac{v_{G S}}{v_{o n}}-1\right)^{2}
\end{aligned}
$$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=0 \mathrm{~V}$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=+3.0 \mathrm{~V}$

Calculate the drain-source current $i_{D S}$ if $v_{D S}=+5.0 \mathrm{~V}$ and $v_{G S}=+6.0 \mathrm{~V}$

## Enhancement-mode MOSFET Circuit

Consider an n-channel enhancement-mode MOSFET circuit with $\mathrm{V}_{\text {on }}=2.0 \mathrm{~V}$,
$\mathrm{K}=0.25 \mathrm{~mA} / \mathrm{V}^{2}, \mathrm{R}_{\mathrm{D}}=1.5 \mathrm{k} \Omega$,
$\mathrm{R}_{1}=4.0 \mathrm{k} \Omega, \mathrm{R}_{2}=6.0 \mathrm{k} \Omega$, and $\mathrm{V}_{\mathrm{DD}}=10 \mathrm{~V}$.
Calculate the operating point for an input voltage $v_{i n}=0 \mathrm{~V}$.


Note that $\mathrm{i}_{\mathrm{G}}=0$. Then, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ form a voltage divider.
$\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\text {in }}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=0 \mathrm{~V}$
Since $\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}<\mathrm{V}_{\text {on }}=2.0 \mathrm{~V}$, the current $\mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A}$.
The load line is $\mathrm{v}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{d}}=10 \mathrm{~V}-0=10 \mathrm{~V}$.
The operating point is

$$
v_{D S}=10 \mathrm{~V} \text { and } \mathrm{i}_{\mathrm{DS}}=0 \mathrm{~A} .
$$

Note that a low input produces a high output.

Calculate the operating point for an input voltage $v_{i n}=10 \mathrm{~V}$.
Note that $\mathrm{i}_{\mathrm{G}}=0$. Then, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ form a voltage divider.
$\mathrm{v}_{\mathrm{GS}}=\mathrm{v}_{\text {in }}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=10[6 /(4+6)]=6.0 \mathrm{~V}$
Assuming saturation, $\mathrm{i}_{\mathrm{DS}}=\mathrm{KV}_{\mathrm{on}}{ }^{2}\left(\mathrm{VGS} / \mathrm{V}_{\text {on }}-1\right)^{2}=(0.25 \mathrm{~mA})(2)^{2}($

$$
\mathrm{i}_{\mathrm{DS}}=(0.25 \mathrm{~mA})(2)^{2}[(6 / 2)-1]^{2}=4.0 \mathrm{~mA}
$$

The load line is $\mathrm{V}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{d}}=10 \mathrm{~V}-(0.004)(1500)=4.0 \mathrm{~V}$.
Note that the inequality for saturation is satisfied

$$
\mathrm{V}_{\mathrm{DS}}-\mathrm{V}_{\mathrm{GS}}=4-6=-2 \mathrm{~V} \geq-\mathrm{V}_{\mathrm{on}}=-2 \mathrm{~V}
$$

The operating point is

$$
\mathrm{v}_{\mathrm{DS}}=4.0 \mathrm{~V} \text { and } \mathrm{i}_{\mathrm{DS}}=4.0 \mathrm{~mA} .
$$

A different approach is to calculate the Thevenin circuit for the input.
Then, $\mathrm{V}_{\mathrm{GG}}=\mathrm{v}_{\mathrm{Th}}=\mathrm{v}_{\text {in }}\left[\mathrm{R}_{2} /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]=6.0 \mathrm{~V}$.
Also, $\mathrm{R}_{\mathrm{Th}}=\left(\mathrm{R}_{1} \mathrm{R}_{2}\right) /\left(\mathrm{R}_{1}+\mathrm{R}_{2}\right)=\mathrm{R}_{\mathrm{g}}$. $\left(\right.$ This does not matter since $\mathrm{i}_{\mathrm{G}}=0$.)
Note that a high input produces a low output.

## Enhancement-mode MOSFET Circuit

Consider an n-channel enhancement-mode MOSFET circuit with $\mathrm{V}_{\text {on }}=2.0 \mathrm{~V}$, $\mathrm{K}=0.25 \mathrm{~mA} / \mathrm{V}^{2}, \mathrm{R}_{\mathrm{D}}=1.5 \mathrm{k} \Omega$, $\mathrm{R}_{1}=4.0 \mathrm{k} \Omega, \mathrm{R}_{2}=6.0 \mathrm{k} \Omega$, and $\mathrm{V}_{\mathrm{DD}}=10 \mathrm{~V}$.

Calculate the operating point for an input voltage $v_{i n}=0 \mathrm{~V}$.


Calculate the operating point for an input voltage $v_{i n}=10 \mathrm{~V}$.

## MOSFET Circuit as an Active Load

Consider a n-channel depletion-mode
MOSFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}$. (Note that $\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}$.) and $\quad V_{D S}=V_{D D}-V_{S S}$

Calculate the current $i_{D S}$ for which the MOSFET circuit has
$v_{D S}=V_{D D}-V_{S S}=0.5 V_{p o}$.


Since $\mathrm{V}_{\mathrm{DS}}=0.5 \mathrm{~V}_{\mathrm{po}}<\mathrm{V}_{\mathrm{po}}$, the transistor is not saturated and
$\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left[2\left(1+\mathrm{V}_{\mathrm{GS}} / \mathrm{V}_{\mathrm{po}}\right)\left(\mathrm{V}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)-\left(\mathrm{V}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}\right]$ for $\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}$.
Then,
$\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left[2(1+0)\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)-\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}\right]$
$\mathrm{i}_{\mathrm{DS}}=(1.0 \mathrm{~mA})\left[2(1)(0.5)-(0.5)^{2}\right]=0.75 \mathrm{~mA}$

Calculate the current $i_{D S}$ for which the MOSFET
 circuit has $v_{D S}=V_{D D}-V_{S S}=V_{p o}$.

Since $\mathrm{v}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{po}}$, the transistor is at the threshold of saturation.
Both equations apply.
Using $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left[2\left(1+\mathrm{vGS} / \mathrm{V}_{\mathrm{po}}\right)\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)-\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}\right]$ for $\mathrm{VGS}_{\mathrm{GS}}=0 \mathrm{~V}$.
Then, $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left[2\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)-\left(\mathrm{v}_{\mathrm{DS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}\right]=(1.0 \mathrm{~mA})\left[2(1)-(1)^{2}\right]=1.0 \mathrm{~mA}$
Using $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DsS}}\left(1+\mathrm{VGS}_{\mathrm{GS}} / \mathrm{V}_{\mathrm{po}}\right)^{2}$ for $\mathrm{VGS}_{\mathrm{GS}}=0 \mathrm{~V}$.
Then, $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{Dss}}(1+0)=1.0 \mathrm{~mA}$

Calculate the current $i_{D S}$ for which the MOSFET circuit has
$v_{D S}=V_{D D}-V_{S S}=2 V_{p o}$.

Since $\mathrm{v}_{\mathrm{DS}}=2 \mathrm{~V}_{\mathrm{po}}>\mathrm{V}_{\mathrm{po}}$, the transistor is in saturation and $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}$ for $\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}$.
Then, $\mathrm{i}_{\mathrm{DS}}=1.0 \mathrm{~mA}$

## MOSFET Circuit as an Active Load

Consider a n-channel depletion-mode MOSFET circuit with $\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA} . \quad\left(\right.$ Note that $\left.\mathrm{V}_{\mathrm{GS}}=0 \mathrm{~V}.\right)$ and $V_{D S}=V_{D D}-V_{S S}$

Calculate the current $i_{D S}$ for which the MOSFET circuit has

$$
v_{D S}=V_{D D}-V_{S S}=0.5 V_{p o} .
$$



Calculate the current $i_{D S}$ for which the MOSFET circuit has $v_{D S}=V_{D D}-V_{S S}=V_{p o}$.

Calculate the current $i_{D S}$ for which the MOSFET circuit has

$$
v_{D S}=V_{D D}-V_{S S}=2 V_{p o} .
$$

## MOSFET Inverter Circuit with an Active Load

Consider the inverter circuit shown with supply voltage $\mathrm{V}_{\mathrm{DD}}=5.00 \mathrm{~V}$. The enhancement-mode MOSFET has
$\mathrm{V}_{\text {on }}=0.50 \mathrm{~V}$ and $\mathrm{K}=1.00 \mathrm{~mA} / \mathrm{V}^{2}$ and the depletion-mode MOSFET has $\mathrm{V}_{\mathrm{po}}=2.00 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{DSS}}=1.50 \mathrm{~mA}$.

Sketch the load-line (LL) on the $i_{D S}$ for $v_{D S I}$ family of curves and calculate the load-line intercepts.

LL KVL is $-\mathrm{V}_{\mathrm{DD}}+\mathrm{V}_{\mathrm{DS} 2}+\mathrm{V}_{\mathrm{DS} 1}=0$
When $\mathrm{i}_{\mathrm{DS}}=0$, the depletion-mode MOSFET has $\mathrm{v}_{\mathrm{DS} 2}=0$. Then the LL gives $\mathrm{v}_{\mathrm{DS} 1}$ as $\mathrm{V}_{\mathrm{DS} 1}=\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{DS} 2}=\mathrm{V}_{\mathrm{DD}}-0=\mathrm{V}_{\mathrm{DD}}=5.00 \mathrm{~V}$

## LL Voltage-axis intercept: ( $V_{\mathrm{Dd}}, 0 \mathrm{~mA}$ ) or ( $5.00 \mathrm{~V}, 0 \mathrm{~mA}$ )

When $\mathrm{v}_{\mathrm{DS} 1}=0$, the maximum current for the active load is $\mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}=1.5 \mathrm{~mA}$.
Note that $i_{D S}=i_{D S 1}=i_{D S 2}$. Then,

## LL Current-axis intercept: <br> ( $0 \mathbf{V}, \mathbf{I}_{\text {Dss }}$ ) or ( $\mathbf{0}, \mathbf{1 . 5 0} \mathbf{m A}$ )



Determine the region of operation (saturated or unsaturated) for the enhancementmode MOSFET for $V_{i}=0 V$ and $V_{i}=4.00 \mathrm{~V}$.

If $\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS} 1}=0 \mathrm{~V}<\mathrm{V}_{\mathrm{ON}}$, then $\mathrm{i}_{\mathrm{DS}}=\mathrm{i}_{\mathrm{DS} 1}=\mathrm{i}_{\mathrm{DS} 2}=0$. The operating point is the voltage-axis LL intercept.

## Saturated Operation

If $\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{GS} 1}=4.00 \mathrm{~V}\left(\mathrm{~V}_{\mathrm{i}}>\mathrm{V}_{\mathrm{ON}}\right)$, then saturated operating conditions are checked. $\mathrm{i}_{\mathrm{DS} 1}=\mathrm{KV}_{\text {on }}{ }^{2}\left(-1+\mathrm{v}_{\mathrm{GS} 1} / \mathrm{V}_{\text {on }}\right)^{2}=1.75 \mathrm{~mA}>1.5 \mathrm{~mA}=\mathrm{I}_{\text {DSS }}=\mathrm{i}_{\text {DSIMAX }}$. Not possible!
(The operating point is near the current-axis LL intercept.)
Hence, $\mathrm{I}_{\mathrm{DSS}}=1.50 \mathrm{~mA}$ and $\mathrm{V}_{\mathrm{DS} 1}$ is the solution of $\mathrm{i}_{\mathrm{DS} 1}=\mathrm{KV}_{\text {on }}{ }^{2}\left[2\left(-1+\mathrm{V}_{\mathrm{GS}} / \mathrm{V}_{\text {on }}\right)\left(\mathrm{VDSI} / \mathrm{V}_{\text {on }}\right)-\left(\mathrm{V}_{\mathrm{DS}} / \mathrm{V}_{\text {on }}\right)^{2}\right]$ for

## Unsaturated Operation

The solution of $1.5 \mathrm{~mA}=(1)(0.5)^{2}\left[2(-1+4 / 0.5)\left(\mathrm{v}_{\mathrm{DSI}} / 0.5\right)-\left(\mathrm{v}_{\mathrm{DSI}} / 0.5\right)^{2}\right]$ is $\mathrm{v}_{\mathrm{DS} 1}=0.2213 \mathrm{~V}$. Note that $\mathrm{V}_{\mathrm{DS} 1}-\mathrm{V}_{\mathrm{GS} 1}=-3.778<-\mathrm{V}_{\mathrm{ON}}=-0.5 \mathrm{~V}$; the inequality is not satisfied for saturated operation.

## MOSFET Inverter Circuit with an Active Load

Consider the inverter circuit shown with supply voltage $\mathrm{V}_{\mathrm{DD}}=5.00 \mathrm{~V}$. The enhancement-mode MOSFET has
$\mathrm{V}_{\text {on }}=0.50 \mathrm{~V}$ and $\mathrm{K}=1.00 \mathrm{~mA} / \mathrm{V}^{2}$ and the depletion-mode MOSFET has

$$
\mathrm{V}_{\mathrm{po}}=2.00 \mathrm{~V} \text { and } \mathrm{I}_{\mathrm{DSS}}=1.50 \mathrm{~mA} .
$$

Sketch the load-line (LL) on the $i_{D S}$ for $v_{D S I}$ family of curves and calculate the load-line intercepts.


Determine the region of operation (saturated or unsaturated) for the enhancementmode MOSFET for $V_{i}=0 \mathrm{~V}$ and $V_{i}=4.00 \mathrm{~V}$.

## Source Follower JFET Circuit Example

Consider a n-channel JFET circuit with
$\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}, \mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{R}_{\mathrm{s}}=1000 \Omega$, $\mathrm{V}_{\mathrm{GG}}=-2.5 \mathrm{~V}$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.
(Note that $\mathrm{V}_{\mathrm{GS}} \neq \mathrm{V}_{\mathrm{GG}}$.)

Calculate the operating point $v_{D S}$ and $i_{D S}$.


Analysis for Operating Point ( $\mathrm{v}_{\mathrm{DS}}, \mathrm{i}_{\mathrm{DS}}$ ).
Kirchhoff's-Voltage-Law on Gate-Source Side:

$$
-\mathrm{V}_{\mathrm{GG}}+\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}+\mathrm{V}_{\mathrm{GS}}=0 \quad \text { or } \quad \mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}
$$

and for operation in the saturation region

$$
-\mathrm{V}_{\mathrm{po}}<\mathrm{VGS}_{\mathrm{GS}}=\left(\mathrm{V}_{\mathrm{GG}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{S}}\right)<0 \text { and } \mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left(1+\mathrm{VGS}_{\mathrm{GS}} / \mathrm{V}_{\mathrm{po}}\right)^{2} .
$$

The simultaneous solution is

$$
\begin{aligned}
& \mathrm{i}_{\mathrm{DS}}=\mathrm{I}_{\mathrm{DSS}}\left(1+\mathrm{vGS} / \mathrm{V}_{\mathrm{po}}\right)^{2}=\mathrm{I}_{\mathrm{DSS}}\left[1+\left(\mathrm{V}_{\mathrm{GG}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}\right) / \mathrm{V}_{\mathrm{po}}\right]^{2} . \\
& \mathrm{i}_{\mathrm{DS}}=(0.001)\left[1+\left(-2.5-1000 \mathrm{i}_{\mathrm{iDS}}\right) /(5)\right]^{2}=(0.001)\left[+0.5-200 \mathrm{i}_{\mathrm{DS}}\right]^{2} \\
& 40,000 \mathrm{i}_{\mathrm{Ds}}{ }^{2}+(-1000-200) \mathrm{i}_{\mathrm{DS}}+0.25=0 \\
& 40,000 \mathrm{i}_{\mathrm{DS}}{ }^{2}+(-1200) \mathrm{i}_{\mathrm{DS}}+0.25=0 \\
& \mathrm{i}_{\mathrm{DS}}{ }^{2}+(-0.03) \mathrm{idS}^{2}+0.00000625=0 \\
& \mathrm{i}_{\mathrm{DS}}=0.02979 \mathrm{~A}, 0.000209 \mathrm{~A}
\end{aligned}
$$

Kirchhoff's-Voltage-Law on Drain Side (the Load-Line Equation):
$-V_{D D}+i_{D S} R_{s}+v_{D S}=0 \quad$ or $\quad V_{D S}=V_{D D}-i_{D S} R_{s}$
For $\mathrm{i}_{\mathrm{DS}}=0.02979 \mathrm{~A}$,
$\mathrm{V}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}=15-29.79=-14.79 \mathrm{~V}$
$\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}=-2.5-29.79=-32.29$
(Does not satisfy needed voltage ranges - not the correct solution)
For ids $=0.00209 \mathrm{~A}$,
$\mathrm{V}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}=15-0.209=14.79 \mathrm{~V}$
$\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{GG}}-\mathrm{i}_{\mathrm{DS}} \mathrm{R}_{\mathrm{s}}=-2.5-0.209=-2.709 \mathrm{~V}$
(Satisfies needed voltage ranges - the correct solution)
Operating point

$$
\mathrm{v}_{\mathrm{DS}}=14.79 \mathrm{~V} \text { and } \mathrm{i}_{\mathrm{DS}}=0.000209 \mathrm{~A}=0.209 \mathrm{~mA}
$$

## Source Follower JFET Circuit Example

Consider a n-channel JFET circuit with
$\mathrm{V}_{\mathrm{po}}=5.0 \mathrm{~V}, \mathrm{I}_{\mathrm{DSS}}=1.0 \mathrm{~mA}, \mathrm{R}_{\mathrm{s}}=1000 \Omega$, $\mathrm{V}_{\mathrm{GG}}=-2.5 \mathrm{~V}$, and $\mathrm{V}_{\mathrm{DD}}=15 \mathrm{~V}$.
(Note that $\mathrm{V}_{\mathrm{GS}} \neq \mathrm{V}_{\mathrm{GG}}$.)

Calculate the operating point $v_{D S}$ and $i_{D S}$.


# EE 2200 INTRO. TO ELECTRONIC DEVICES 

## ECE, Missouri S\&T

© 2023 Steve E. Watkins

# WORKSHEETS <br> FOR <br> EXAMINATION 3 

Topics Include:
Ideal and Non-Ideal OpAmp Models
OpAmp Circuits
Optoelectronics
Laser Diodes and Photodiodes and Related Circuits

| Boltzmann's constant: | $\mathrm{k}=1.381 \times 10^{-23} \mathrm{~J} / \mathrm{K}=8.618 \times 10^{-5} \mathrm{eV} / \mathrm{K}$ |  |  |
| :--- | :--- | :--- | :--- |
| Planck's constant | $\mathrm{h}=4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}=6.626 \times 10^{-34} \mathrm{~J}$-sec |  |  |
| Electronic charge: | $\mathrm{q}=1.602 \times 10^{-19} \mathrm{C}$ |  |  |
| kT at 300 K | $\mathrm{kT}=0.0259 \mathrm{eV}$ | eV-J conversion | $1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$ |
| Free-space permittivity | $\varepsilon_{\mathrm{O}}=8.854 \times 10^{-14} \mathrm{~F} / \mathrm{cm}$ | Speed of Light | $\mathrm{c}=2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s}$ |
| Relative permittivity | Si: 11.9 | Ge: 16.0 | GaAs: 13.1 |
| Bandgap energies | Si: 1.12 eV | Ge: 0.67 eV | GaAs: 1.42 eV |

## INVERTING OPAMP WITH FINITE GAIN

Consider an inverting OpAmp circuit with an OpAmp gain of $\mathrm{A}, \mathrm{V}_{\mathrm{S}}=+5 \mathrm{~V}$, $\mathrm{R}_{\mathrm{a}}=500 \Omega$, and $\mathrm{R}_{\mathrm{b}}=1,000 \Omega$. Besides the OpAmp gain $A$, the OpAmp is ideal.


Calculate the output voltage $V_{0}$, if $A$ goes to infinity.
Kirchhoff's-Current-Law for the (-) terminal:
(Note that the OpAmp input current is zero)

$$
\begin{aligned}
& \left(-\Delta v-v_{s}\right) / R_{a}+\left(-\Delta v-v_{o}\right) / R_{b}=0 \quad \text { and } \quad \Delta v=v_{o} / A \\
& \left(-v_{o} / A-v_{s}\right) / R_{a}+\left(-v_{0} / A-v_{o}\right) / R_{b}=0
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& \mathrm{vs}_{\mathrm{s}} / \mathrm{R}_{\mathrm{a}}=\left(-1 / \mathrm{AR}_{\mathrm{a}}-1 / \mathrm{AR} R_{\mathrm{b}}-1 / \mathrm{R}_{\mathrm{b}}\right) \mathrm{v}_{\mathrm{o}}=(-1 / \mathrm{A})\left[\left(\mathrm{R}_{\mathrm{b}}+\mathrm{R}_{\mathrm{a}}+\mathrm{AR} \mathrm{R}_{\mathrm{a}}\right) / \mathrm{R}_{\mathrm{a}} \mathrm{R}_{\mathrm{b}}\right] \mathrm{v}_{\mathrm{o}} \\
& \mathrm{v}_{\mathrm{o}}=-\left\{\mathrm{R}_{\mathrm{b}} /\left[\left(\mathrm{R}_{\mathrm{b}}+\mathrm{R}_{\mathrm{a}}\right) / \mathrm{A}+\mathrm{R}_{\mathrm{a}}\right]\right\} \mathrm{v}_{\mathrm{s}} \\
& \lim _{\mathrm{A} \text { to } \infty} \mathrm{V}_{\mathrm{o}}=-\left(\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{a}}\right) \mathrm{v}_{\mathrm{S}}
\end{aligned}
$$

In the limit

$$
v_{o}=-\left(R_{b} / R_{a}\right) v_{s}=-(1000 / 500)(5 \mathrm{~V})=-10 \mathrm{~V}
$$

Calculate the output voltage $V_{0}$, if $A$ equals 10,000 .
With a finite A , the output is

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{o}}=-\left\{\mathrm{R}_{\mathrm{b}} /\left[\left(\mathrm{R}_{\mathrm{b}}+\mathrm{R}_{\mathrm{a}}\right) / \mathrm{A}+\mathrm{R}_{\mathrm{a}}\right]\right\} \mathrm{v}_{\mathrm{s}} \\
& \mathrm{v}_{\mathrm{o}}=-\{1000 /[(1000+500) / 10,000+500]\}(5 \mathrm{~V}) \\
& \mathrm{v}_{\mathrm{o}}=-9.9970 \mathrm{~V}
\end{aligned}
$$

Calculate the output voltage $V_{0}$, if A equals 100,000.
With a finite $A$, the output is

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{o}}=-\left\{\mathrm{R}_{\mathrm{b}} /\left[\left(\mathrm{R}_{\mathrm{b}}+\mathrm{R}_{\mathrm{a}}\right) / \mathrm{A}+\mathrm{R}_{\mathrm{a}}\right]\right\} \mathrm{vs}_{\mathrm{s}} \\
& \mathrm{v}_{\mathrm{o}}=-\{1000 /[(1000+500) / 100,000+500]\}(5 \mathrm{~V}) \\
& \mathrm{v}_{\mathrm{o}}=-9.9997 \mathrm{~V}
\end{aligned}
$$

## INVERTING OPAMP WITH FINITE GAIN

Consider an inverting OpAmp circuit with an OpAmp gain of $\mathrm{A}, \mathrm{V}_{\mathrm{s}}=+5 \mathrm{~V}$, $\mathrm{R}_{\mathrm{a}}=500 \Omega$, and $\mathrm{R}_{\mathrm{b}}=1,000 \Omega$. Besides the OpAmp gain $A$, the OpAmp is ideal.


Calculate the output voltage $V_{0}$, if $A$ goes to infinity.

Calculate the output voltage $V_{0}$, if A equals 10,000.

Calculate the output voltage Vo, if A equals 100,000.

## INVERTING OPAMP EXAMPLE

Consider an inverting OpAmp circuit with $\mathrm{R}_{\mathrm{a}}=1,000 \Omega, \mathrm{R}_{\mathrm{b}}=3,000 \Omega$, and $\mathrm{V}_{\mathrm{s}}=+5.0 \mathrm{~V}$, Assume that the OpAmp is ideal.


Calculate the currents through the $R_{a}$ resistor and through the $R_{b}$ resistor.
By the prior analysis, the voltage $\Delta \mathrm{v}$ is driven to zero. Hence,
For $\mathrm{R}_{\mathrm{a}}, \mathrm{i}_{\mathrm{a}}=(-\Delta \mathrm{v}-\mathrm{vs}) / \mathrm{R}_{\mathrm{a}}=(0-5) /(1,000)=-0.005 \mathrm{~A}=-5.0 \mathrm{~mA}$
For $\mathrm{R}_{\mathrm{b}}$, the output voltage must be known, The prior analysis gives (in the limit)

$$
v_{o}=-\left(R_{b} / R_{a}\right) v_{S}=-(3000 / 1000)(5 \mathrm{~V})=-15 \mathrm{~V}
$$

Then, $\mathrm{i}_{\mathrm{b}}=\left(-\Delta \mathrm{v}-\mathrm{v}_{\mathrm{o}}\right) / \mathrm{R}_{\mathrm{b}}=(0+15) /(3,000)=0.005 \mathrm{~A}=5.0 \mathrm{~mA}$

Calculate the current generated by the OpAmp if a load resistor of $R_{L}=1,000 \Omega$ is attached.

As before, the output voltage is

$$
\mathrm{v}_{\mathrm{o}}=-\left(\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{a}}\right) \mathrm{v}_{\mathrm{s}}=-(3000 / 1000)(5 \mathrm{~V})=-15 \mathrm{~V}
$$

The load current is

$$
\mathrm{i}_{\mathrm{L}}=\left(\mathrm{v}_{\mathrm{o}}\right) / \mathrm{R}_{\mathrm{L}}=(-15) /(1,000)=-0.015 \mathrm{~A}=-15.0 \mathrm{~mA}
$$

The current through the resistor network is

$$
\mathrm{i}_{\mathrm{b}}=(-\Delta \mathrm{v}-\mathrm{vs}) / \mathrm{R}_{\mathrm{b}}=(0+15) /(3,000)=0.005 \mathrm{~A}=5.0 \mathrm{~mA}
$$

The total current from the OpAmp is
$\mathrm{i}_{\text {Total }}=\mathrm{i}_{\mathrm{L}}-\mathrm{i}_{\mathrm{b}}=-15 \mathrm{~mA}-5.0 \mathrm{~mA}=-20 \mathrm{~mA}$

## INVERTING OPAMP EXAMPLE

Consider an inverting OpAmp circuit with $\mathrm{R}_{\mathrm{a}}=1,000 \Omega, \mathrm{R}_{\mathrm{b}}=3,000 \Omega$, and $\mathrm{V}_{\mathrm{s}}=+5.0 \mathrm{~V}$, Assume that the OpAmp is ideal.


Calculate the currents through the $R_{\text {a }}$ resistor and through the $R_{b}$ resistor.

Calculate the current generated by the OpAmp if a load resistor of $R_{L}=1,000 \Omega$ is attached.

## NON-INVERTING OPAMP EXAMPLE

Consider a non-inverting OpAmp circuit with $\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\text {Total }}=1.0 \mathrm{k} \Omega$ with $\mathrm{R}_{\mathrm{a}}=\mathrm{w} \mathrm{R}_{\text {Total }}$ and $\mathrm{R}_{\mathrm{b}}=(1-\mathrm{w}) \mathrm{R}_{\text {Total }}$. Let $\mathrm{V}_{\mathrm{S}}=+10.0 \mathrm{mV}$. Assume that the OpAmp is ideal.


For a gain of 2, calculate the fraction $w$ and the resistor values for $R_{a}$ and $R_{b}$.
Note that $\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\text {Total }}$ where $0<\mathrm{w}<1$
The solution for this configuration is

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{o}} / \mathrm{vs}_{\mathrm{s}}=\left(\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}\right) / \mathrm{R}_{\mathrm{a}}=1+\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{a}}=1+\left[(1-\mathrm{w}) \mathrm{R}_{\text {Total }} / \mathrm{w} \mathrm{R}_{\text {Total }}\right]= \\
& \mathrm{v}_{\mathrm{o}} / \mathrm{vs}=1+\left[(1-\mathrm{w}) \mathrm{R}_{\text {Total }} / \mathrm{w} \mathrm{R}_{\text {Total }}\right]=1+1 / \mathrm{w}-1=1 / \mathrm{w}
\end{aligned}
$$

For $2=1 / w$, then $w=1 / 2$.
Then, $\mathrm{R}_{\mathrm{a}}=\mathrm{wR}_{\text {Total }}=(1 / 2) 1.0 \mathrm{k} \Omega=0.5 \mathrm{k} \Omega$
and $\mathrm{R}_{\mathrm{b}}=(1-\mathrm{w}) \mathrm{R}_{\text {Total }}=(1 / 2) 1.0 \mathrm{k} \Omega=0.5 \mathrm{k} \Omega$

Calculate the power dissipated in $R_{a}$ and $R_{b}$.
The output voltage is

$$
\mathrm{v}_{\mathrm{o}}=\left\{1+\left[(1-\mathrm{w}) \mathrm{R}_{\text {Total }} / \mathrm{wR}_{\text {Total }}\right]\right\} \mathrm{vs}=(1 / \mathrm{w}) \mathrm{vs}=2(10 \mathrm{mV})=20 \mathrm{mV}
$$

Then, the current through $\mathrm{R}_{\text {Total }}$ is

$$
\mathrm{I}_{\text {Total }}=\left(\mathrm{v}_{\mathrm{o}}\right) /\left(\mathrm{R}_{\mathrm{a} \text { Total }}\right)=(0.020 \mathrm{~V}) /(1.0 \mathrm{k} \Omega)=0.020 \mathrm{~mA}
$$

And

$$
\mathrm{P}_{\mathrm{a}}=\mathrm{P}_{\mathrm{b}}=\left(\mathrm{R}_{\mathrm{a}, \mathrm{~b}}\right)\left(\mathrm{I}_{\text {Total }}\right)^{2}=(0.5 \mathrm{k} \Omega)(0.020 \mathrm{~mA})^{2}=0.20 \mu \mathrm{~W}
$$

The total power dissipated in $\mathrm{R}_{\mathrm{a}}$ and $\mathrm{R}_{\mathrm{b}}$ is

$$
\mathrm{P}_{\text {Total }}=\mathrm{P}_{\mathrm{a}}+\mathrm{P}_{\mathrm{b}}=0.40 \mu \mathrm{~W}
$$

## NON-INVERTING OPAMP EXAMPLE

Consider a non-inverting OpAmp circuit with $\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\text {Total }}=1.0 \mathrm{k} \Omega$ with $\mathrm{R}_{\mathrm{a}}=w \mathrm{R}_{\text {Total }}$ and $\mathrm{R}_{\mathrm{b}}=(1-\mathrm{w}) \mathrm{R}_{\text {Total }}$. Let $\mathrm{V}_{\mathrm{S}}=+10.0 \mathrm{mV}$. Assume that the OpAmp is ideal.


For a gain of 2, calculate the fraction w and the resistor values for $R_{a}$ and $R_{b}$.

Calculate the power dissipated in $R_{a}$ and $R_{b}$.

## SUBTRACTOR OPAMP EXAMPLE

Consider a subtractor OpAmp circuit with $\mathrm{R}_{\mathrm{a}}=(\mathrm{x}) 1,000 \Omega, \mathrm{R}_{\mathrm{c}}=(1-\mathrm{x}) 1,000 \Omega$, $\mathrm{R}_{\mathrm{b}}=(\mathrm{y}) 100 \Omega$, and $\mathrm{R}_{\mathrm{d}}=(1-\mathrm{y}) 100 \Omega$ where $0<x<1$ and $0<y<1$. Assume that the OpAmp is ideal.


Calculate the values $x$ and $y$ for which the magnitude of the component gain for input $V_{S a}$ is 3 and the magnitude of the component gain for input $V_{S b}$ is also 3 .

In the limit, the analysis gives

$$
\mathrm{v}_{\mathrm{o}}=\mathrm{v}_{\mathrm{oa}}+\mathrm{v}_{\mathrm{ob}}=-\left[\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right] \mathrm{V}_{\mathrm{Sa}}+\left\{\left[1+\left(\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right)\right] /\left[1+\left(\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{d}}\right)\right]\right\} \mathrm{V}_{\mathrm{sb}}
$$

Hence, for input $\mathrm{V}_{\mathrm{sa}}$

$$
\begin{aligned}
& \left|\mathrm{V}_{\mathrm{oa}} / \mathrm{V}_{\mathrm{Sa}}\right|=\left[\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right]=[(1-\mathrm{x})(1000)] /[(\mathrm{x})(1000)]=(1 / \mathrm{x}-1)=3 \\
& \mathrm{x}=0.25
\end{aligned}
$$

Then, $\mathrm{R}_{\mathrm{a}}=250 \Omega$ and $\mathrm{R}_{\mathrm{c}}=750 \Omega$
Hence, for input $\mathrm{V}_{\mathrm{Sa}}$

$$
\begin{aligned}
& \left|\mathrm{V}_{\mathrm{ob}} / \mathrm{V}_{\mathrm{Sb}}\right|=\left\{\left[1+\left(\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right)\right] /\left[1+\left(\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{d}}\right)\right]\right\} \\
& \left|\mathrm{V}_{\mathrm{ob}} / \mathrm{Vsb}\right|=\{[1+(750 / 250)] /[1+(\mathrm{y}) /(1-\mathrm{y})]\}=(4) /[1+1 /(1 / \mathrm{y}-1)] \\
& \left|\mathrm{V}_{\mathrm{ob}} / \mathrm{V}_{\mathrm{Sb}}\right|=(4) /[1+1 /(1 / \mathrm{y}-1)]=3 \\
& 4 / 3=1+1 /(1 / \mathrm{y}-1) \text { or } 4 / 3-1=1 /(1 / \mathrm{y}-1) \\
& 3=1 / \mathrm{y}-1 \\
& \mathrm{y}=0.25
\end{aligned}
$$

Then, $\mathrm{R}_{\mathrm{b}}=25 \Omega$ and $\mathrm{R}_{\mathrm{d}}=75 \Omega$
Calculate the output voltage $V_{0}$, if $V_{S a}=2 \mathrm{~V}$ and $V_{S b}=4 \mathrm{~V}$.
In the limit, the analysis gives

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{o}}=\mathrm{v}_{\mathrm{oa}}+\mathrm{v}_{\mathrm{ob}}=-\left[\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right] \mathrm{V}_{\mathrm{Sa}}+\left\{\left[1+\left(\mathrm{R}_{\mathrm{c}} / \mathrm{R}_{\mathrm{a}}\right)\right] /\left[1+\left(\mathrm{R}_{\mathrm{b}} / \mathrm{R}_{\mathrm{d}}\right)\right]\right\} \mathrm{V}_{\mathrm{sb}} \\
& \mathrm{v}_{\mathrm{o}}=\mathrm{v}_{\mathrm{oa}}+\mathrm{v}_{\mathrm{ob}}=-(3)(2 \mathrm{~V})+(3)(4 \mathrm{~V})=+6 \mathrm{~V} \\
& \mathrm{v}_{\mathrm{o}}=+6 \mathrm{~V}
\end{aligned}
$$

## SUBTRACTOR OPAMP EXAMPLE

Consider a subtractor OpAmp circuit with $\mathrm{R}_{\mathrm{a}}=(\mathrm{x}) 1,000 \Omega, \mathrm{R}_{\mathrm{c}}=(1-\mathrm{x}) 1,000 \Omega$, $\mathrm{R}_{\mathrm{b}}=(\mathrm{y}) 100 \Omega$, and $\mathrm{R}_{\mathrm{d}}=(1-\mathrm{y}) 100 \Omega$ where $0<x<1$ and $0<y<1$. Assume that the OpAmp is ideal.


Calculate the values $x$ and $y$ for which the magnitude of the component gain for input $V_{\text {Sa }}$ is 3 and the magnitude of the component gain for input $V_{\text {sb }}$ is also 3 .

Calculate the output voltage $V_{0,}$, if $V_{S a}=2 V$ and $V_{S b}=4 V$.

## DESIGN EXAMPLE

Design an OpAmp circuit with a minimum number of ideal OpAmps that has the following characteristics:

Has an overall signal gain of - 5 and
Draws no current from the input source.

## Implementation \#1

Stage 1: A Buffer OpAmp Circuit to give a gain of $\mathrm{v}_{\mathrm{ol}} / \mathrm{vs}_{\mathrm{s}}=+1$ while drawing no current from the input source.

Stage 2: An Inverting OpAmp Circuit to give a gain of $\mathrm{v}_{02} / \mathrm{v}_{01}=-5$. One possible set of resistors is $R_{a}=200 \Omega$ and $R_{b}=1000 \Omega$, i.e. $-R_{b} / R_{a}=-5$.

The overall gain is
$\left(\mathrm{vor}^{2} / \mathrm{vs}_{\mathrm{s}}\right)=\left(\mathrm{v}_{\mathrm{ol}} / \mathrm{vs}\right)\left(\mathrm{vor}_{\mathrm{o} 2} / \mathrm{vol}_{\mathrm{ol}}\right)$
$\left(\mathrm{v}_{\mathrm{o} 2} / \mathrm{vs}_{\mathrm{s}}\right)=(+1)(-5)=-5$.


## Implementation \#2

Stage 1: A Non-inverting OpAmp Circuit to give a gain of $\mathrm{v}_{\mathrm{ol}} / \mathrm{vs}_{\mathrm{s}}=+5$ (note this circuit draws no current from the input source). One possible set of resistors is $R_{a}=250 \Omega$ and $R_{b}=1000 \Omega$, i.e. $\left(1+R_{b} / R_{a}\right)=+5$.

Stage 2: An Inverting OpAmp Circuit to give a gain of $v_{02} / v_{01}=-1$. One possible set of resistors is $R_{a}=500 \Omega$ and $R_{b}=500 \Omega$, i.e. $-R_{b} / R_{a}=-1$.

The overall gain is
$\left(\mathrm{v}_{0} / / \mathrm{vs}_{\mathrm{s}}\right)=\left(\mathrm{v}_{\mathrm{o} 1} / \mathrm{vs}_{\mathrm{s}}\right)\left(\mathrm{vor}_{\mathrm{o}} / \mathrm{v}_{\mathrm{o} 1}\right)$
$\left(\mathrm{v}_{\mathrm{o} 2} / \mathrm{vs}\right)=(+5)(-1)=-5$.


## OPAMP DESIGN EXAMPLE

Design an OpAmp circuit with a minimum number of ideal OpAmps that has the following characteristics:

Has an overall signal gain of - 5 and Draws no current from the input source.

Implementation \#1

## SEMICONDUCTOR ABSORPTION EXAMPLE

Consider a crystal semiconductor InP. It has an energy gap of $\mathrm{E}_{\mathrm{G}}=1.35 \mathrm{eV}$.

Electron Charge
Speed of Light (Vacuum)
Planck's Constant

$$
\begin{aligned}
& \mathrm{q}=1.602 \times 10^{-19} \mathrm{C} \\
& \mathrm{c}=2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s} \\
& \mathrm{~h}=4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}=6.626 \times 10^{-34} \mathrm{~J}-\mathrm{sec}
\end{aligned}
$$

Calculate the optical wavelength and frequency associated with a bandgap transition.

The photon energy for a wavelength of light $\lambda$ is $E_{p}=h c / \lambda$. A semiconductor will absorb light if the photon energy is greater than or equal to the energy gap $\mathrm{E}_{\mathrm{G}}$. Hence, the maximum wavelength for absorption is
$\lambda=\mathrm{hc} / \mathrm{E}_{\mathrm{G}}=\left(4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}\right)\left(2.998 \times 10^{10} \mathrm{~cm} / \mathrm{sec}\right) /(1.35 \mathrm{eV})$
$\lambda=9.185 \times 10^{-5} \mathrm{~cm}=0.9185 \mu \mathrm{~m}$
$\mathrm{f}=\mathrm{c} / \lambda=\left(2.998 \times 10^{10} \mathrm{~cm} / \mathrm{sec}\right) /\left(9.185 \times 10^{-5} \mathrm{~cm}\right)=3.264 \times 10^{14} \mathrm{~Hz}$
In what part of the optical spectrum is this threshold value?
The infrared portion of the spectrum, or the near infrared.
The divisions of the Infrared spectrum are $0.700 \mu \mathrm{~m}$ to $10^{5} \mu \mathrm{~m}$.

For a 1.0 mW wave at this threshold wavelength, how many photons are present?
The number of photons per second are $\mathrm{P} \lambda / \mathrm{hc}$.
$\mathrm{P} \lambda / \mathrm{hc}=(0.001 \mathrm{~J} / \mathrm{sec})\left(9.185 \times 10^{-5} \mathrm{~cm}\right) /$
$\left(6.626 \times 10^{-34} \mathrm{~J}\right.$-sec) $\left(2.998 \times 10^{10} \mathrm{~cm} / \mathrm{sec}\right)$
$\mathrm{P} \lambda / \mathrm{hc}=\left(4.624 \times 10^{15}\right.$ photons $\left./ \mathrm{sec}\right)$

## SEMICONDUCTOR ABSORPTION EXAMPLE

Consider a crystal semiconductor InP. It has an energy gap of $\mathrm{E}_{\mathrm{G}}=1.35 \mathrm{eV}$.

Electron Charge
Speed of Light (Vacuum)
Planck's Constant
$\mathrm{q}=1.602 \times 10^{-19} \mathrm{C}$
$\mathrm{c}=2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s}$
$\mathrm{h}=4.136 \times 10^{-15} \mathrm{eV}-\mathrm{sec}=6.626 \times 10^{-34} \mathrm{~J}$-sec

Calculate the optical wavelength and frequency associated with a bandgap transition.

In what part of the optical spectrum is this threshold value?

For a 1.0 mW wave at this threshold wavelength, how many photons are present?

## SEMICONDUCTOR ABSORPTION EXAMPLE

A laser beam of irradiance $1.0 \mathrm{~W} / \mathrm{m}^{2}$ is incident upon a silicon ( Si ) sample. The absorption coefficient $\alpha_{\mathrm{L}}$ at the laser wavelength is $1000 \mathrm{~cm}^{-1}$.

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to one half of the initial value, i.e. $0.50 \mathrm{~W} / \mathrm{m}^{2}$. Neglect reflections.

Neglecting reflection, $I=I_{o} \exp \left(-\alpha_{L} x\right)$. For the given conditions,

$$
\begin{aligned}
& 0.50=1.0 \exp \left(-\alpha_{\mathrm{L}} \mathrm{~L}\right) \\
& \mathrm{L}=\left(-1 / \alpha_{\mathrm{L}}\right) \ln (0.50 / 1.0)=\left(-1 / 1000 \mathrm{~cm}^{-1}\right) \ln (0.50)=0.000693 \mathrm{~cm} \\
& \mathrm{~L}=6.93 \mu \mathrm{~m}
\end{aligned}
$$

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to $0.10 \mathrm{~W} / \mathrm{m}^{2}$. Neglect reflections.

Neglecting reflection, $I=I_{o} \exp \left(-\alpha_{L} x\right)$. For the given conditions,

$$
\begin{aligned}
& 0.10=1.0 \exp \left(-\alpha_{\mathrm{L}} \mathrm{~L}\right) \\
& \mathrm{L}=\left(-1 / \alpha_{\mathrm{L}}\right) \ln (0.10 / 1.0)=\left(-1 / 1000 \mathrm{~cm}^{-1}\right) \ln (0.10)=0.00230 \mathrm{~cm} \\
& \mathrm{~L}=23.0 \mu \mathrm{~m}
\end{aligned}
$$

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to $0.010 \mathrm{~W} / \mathrm{m}^{2}$. Neglect reflections.

Neglecting reflection, $I=I_{o} \exp \left(-\alpha_{L} x\right)$. For the given conditions,

$$
\begin{aligned}
& 0.50=1.0 \exp \left(-\alpha_{\mathrm{L}} \mathrm{~L}\right) \\
& \mathrm{L}=\left(-1 / \alpha_{\mathrm{L}}\right) \ln (0.01 / 1.0)=\left(-1 / 1000 \mathrm{~cm}^{-1}\right) \ln (0.01)=0.00461 \mathrm{~cm} \\
& \mathrm{~L}=46.1 \mu \mathrm{~m}
\end{aligned}
$$

## SEMICONDUCTOR ABSORPTION EXAMPLE

A laser beam of irradiance $1.0 \mathrm{~W} / \mathrm{m}^{2}$ is incident upon a silicon ( Si ) sample. The absorption coefficient $\alpha_{\mathrm{L}}$ at the laser wavelength is $1000 \mathrm{~cm}^{-1}$.

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to one half of the initial value, i.e. $0.50 \mathrm{~W} / \mathrm{m}^{2}$. Neglect reflections.

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to $0.10 \mathrm{~W} / \mathrm{m}^{2}$. Neglect reflections.

Calculate the thickness in microns ( $\mu \mathrm{m}$ ) for which the transmitted irradiance is reduced to $0.010 \mathrm{~W}^{2}$. Neglect reflections.

## APD EXAMPLE

A Si avalanche photodiode is reverse-biased for source voltage $\mathrm{V}_{\mathrm{S}}=-80.0 \mathrm{~V}$. The reverse saturation current is 0.010 mA and the $\lambda=900 \mathrm{~nm}$. Assume $|\mathrm{V}| \gg \mathrm{kT} / \mathrm{q}$. The resistance is $\mathrm{R}=20.0 \mathrm{k} \Omega$.


If the optical power is 0.10 mW , calculate the needed quantum efficiency (including the avalanche gain) to produce a diode current of - 1.0 mA .

If the diode current is -1.0 mA , then $\mathrm{I}_{\mathrm{L} \text { ight }}$ is

$$
\begin{aligned}
& \mathrm{I}=\mathrm{I}_{\mathrm{o}}\left[\exp \left(\mathrm{q} \mathrm{~V}_{\mathrm{d}} / \mathrm{kT}\right)-1\right]-\mathrm{I}_{\text {light }}=-(0.010 \mathrm{~mA})-\mathrm{I}_{\text {light }}=-1.0 \mathrm{~mA} \\
& \mathrm{I}_{\text {light }}=-(0.010 \mathrm{~mA})+1.0 \mathrm{~mA}=+0.99 \mathrm{~mA}
\end{aligned}
$$

Then, $\mathrm{I}_{\text {light }}=\eta q \mathrm{P} \lambda / \mathrm{hc}$ or

$$
\begin{aligned}
& \eta=I_{\text {light }} \mathrm{hc} /(\lambda \mathrm{qP}) \\
& \eta=\left(0.99 \times 10^{-3} \mathrm{C} / \mathrm{s}\right)\left(6.626 \times 10^{-34} \mathrm{~J}-\mathrm{s}\right)\left(2.998 \times 10^{10} \mathrm{~cm} / \mathrm{s}\right) / \\
& \quad \quad\left[\left(0.9 \times 10^{-4} \mathrm{~cm}\right)\left(1.602 \times 10^{-19} \mathrm{C}\right)(0.0001 \mathrm{~J} / \mathrm{s})\right] \\
& \eta=I_{\text {light }} \mathrm{hc} /(\lambda \mathrm{qP})=13.6
\end{aligned}
$$

For the diode current of -1.0 mA , calculate the diode voltage.
The circuit load-line equation is

$$
\begin{aligned}
& -\mathrm{V}_{\mathrm{s}}+\mathrm{V}_{\mathrm{d}}+\mathrm{IR}=0 \\
& \mathrm{~V}_{\mathrm{d}}=+\mathrm{V}_{\mathrm{S}}-\mathrm{IR}=(-80)-(-1.0 \mathrm{~mA})(20.0 \mathrm{k} \Omega) \\
& \mathrm{V}_{\mathrm{d}}=-60.0 \mathrm{~V}
\end{aligned}
$$

The operating point is $\mathrm{V}_{\mathrm{d}}=-60.0 \mathrm{~V}$ and $\mathrm{I}=-1.0 \mathrm{~mA}$

## APD EXAMPLE

A Si avalanche photodiode is reverse-biased for source voltage $\mathrm{V}_{\mathrm{S}}=-80.0 \mathrm{~V}$. The reverse saturation current is 0.010 mA and the $\lambda=900 \mathrm{~nm}$. Assume $|\mathrm{V}| \gg \mathrm{kT} / \mathrm{q}$. The resistance is $\mathrm{R}=20.0 \mathrm{k} \Omega$.


If the optical power is 0.10 mW , calculate the needed quantum efficiency (including the avalanche gain) to produce a diode current of -1.0 mA .

For the diode current of - 1.0 mA , calculate the diode voltage.

